Dynamic analysis of dissipative structures undergoing parametric changes

Analyse dynamique des structures dissipatives subissant des modifications paramétriques

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ABSTRACT. In nearly all mechanical constructions, vibrations can gradually damage structures. For that reason, it is imperative to develop a mathematical model that describes these vibrations for the purpose of minimizing their consequences during the design. If we consider the structure's size and complexity, as well as the repetitive nature which characterizing these procedures, an exact reanalysis is a long and expensive option. That being the case, several methods have been developed to help predict a system's dynamic behavior. In this work, we develop a method of Modal Reanalysis, which is precise, efficient, and useful for both conservative and dissipative structures. This technique can also decrease issues related to these structures, and consider the effects resulting from modal truncation. The originality lies in the structure of the obtained formula, where attention is focused on the contribution of the unknown modes. Such contribution can be either calculable for a finite element model, or identifiable from experiment model tests.
KEYWORDS. Dissipative structure, Finite element method, Parametric modification, Reanalysis, Static residual matrix, Truncated modal basis, Vibration.

1. Introduction

The prediction of the dynamic behaviour of a mechanical structure is a requirement for the insurance of the equipment's reliability and the respect for vibratory criteria. To reach these objectives, various potential prototypes and their respective models are developed before a choice is made. Indeed, when a classic method is used in order to define the eigensolutions and the dynamic responses of the initial structure, the model should be reviewed and the eigensolutions must be recalculated within the same process. This operation is repeated, for each modification of the structure, until obtaining the desired product. To speed up and to optimize the calculations, reanalysis methods [1–3], which employ existing data as the basis for estimating the behaviour of a design variants, were used. An exact reanalysis might be considered as the most direct approach, but the large sizes of the discrete models (several thousand degrees of freedom), frequently encountered, make this strategy both costly and burdensome. To avoid these difficulties, many research groups opt for the approximate reanalysis technique. This approach consists of condensing [4] the modified model using a truncated basis of eigenvectors associated with the initial model [5–7]. The difficulty of each of these procedures lies in how to reduce truncation effects. The latter may affect the accuracy of the predicted dynamic behaviour of the modified structure. The effects resulting from modal truncation have already been discussed by several authors [7–11]. In order to reduce these effects, S. Cogan et al. [8], H. Ait rimouch et al. [12], Y. M. Ram et al. [13] and A. Bouazzouni et al. [6] have proposed many strategies to enrich the sub-basis modal using residual static vectors and have succeeded in improving the precision of the solution. A. B. Palazzo-lo et al. [14] proposed the Receptance approach,

usable for systems with large and multiple modifications but only used for undamped structures. Other authors [15–17] presented the effect of the residual terms in a more or less approximate form. Moreover, most of these reanalyses techniques were only used in the case of non-dissipative systems and few of them were considered as the damping in these systems. On the other hand, even though the damping was introduced in the theorical studies [6], [17], it was validated just in conservative structures. In this work, a reanalysis method is proposed using data obtained either from simulation or from the theoretical analysis based on a finite element model of the structure. The novelty of this method lies in the fact that the strategy takes into account, in a very significant manner, the contribution of the unknown modes. This contribution will be represented by the residual flexibility terms of both conservative and dissipative structures with non-proportional damping, which can be determined theoretically; the obtained results were validated by Matlab simulations.

2. General assumptions

In the frequency domain, the autonomous mechanic behavior equation of a dissipative structure can be expressed as follows:

$$(s_\nu^2 M + s_\nu B + K) z_\nu = 0 \tag{1}$$

where K,M, and $B \in \mathbb{R}^{n,n}$ are respectively the stiffness, the mass and the damping matrices, which are real, symmetric and positive definite, n is the total number of degrees of freedom (DOFs); $s_\nu \in \mathbb{C}^{n,n}$ and $z_\nu \in \mathbb{C}^{n,n}$ are the ν^{th} eigenvalue and eigenvector of the dissipative structure, respectively. The modified structure is characterized by the mass matrix $\hat{M} = M + \Delta M \in \mathbb{R}^{n,n}$, stiffness matrix $\hat{K} = K + \Delta K \in \mathbb{R}^{n,n}$ and the damping matrix $\hat{B} = B + \Delta B \in \mathbb{R}^{n,n}$. where ΔM,ΔK, and ΔB are the mass, the stiffness, and the damping modification matrices respectively. The modification structure will have the following form:

$$(\hat{s}_\nu^2 \hat{M} + \hat{s}_\nu \hat{B} + \hat{K}) \hat{z}_\nu = 0 \tag{2}$$

We are trying to determine eigensolutions $\hat{s}_\nu \in \mathbb{C}^{n,n}$ and $\hat{z}_\nu \in \mathbb{C}^{n,n}$ of the M.S, knowing the modification introduced and the modal parameters identified from the original structure.

3. Conservative structures

Consider a conservative self-adjoint mechanical structure ($B = 0$), represented by mass and stiffness matrices M and $K \in \mathbb{R}^{n,n}$, respectively, both symmetric and positive definite; n is the total number of degrees of freedom (DOFs) of the discrete structural model. Its eigensolutions are regrouped in the modal $X = [\ldots\ x_\nu\ \ldots] \in \mathbb{R}^{n,n}$ and the spectral $\Lambda = diag\{\lambda_\nu\} \in \mathbb{R}^{n,n}$) matrices, $\nu = 1, \ldots, n$, which satisfy the bi-orthogonal relations:

$$X^T M X = I;\ X^T K X = \Lambda \tag{3}$$

where I is the identity matrix. The eigenvalues are ordered by magnitude ($\lambda_1 \leq \ldots \leq \lambda_n$). The modified structure is characterized by the mass matrix $\hat{M} = M + \Delta M \in \mathbb{R}^{n,n}$, the stiffness matrix $\hat{K} = K + \Delta K \in \mathbb{R}^{n,n}$ matrices, where ΔM and ΔK are the mass, the stiffness modification matrices, respectively. Its eigenvalue problem, in physical coordinates, is given by:

$$(\hat{K} - \hat{\lambda}_\nu \hat{M}) \hat{x}_\nu = 0;\ \nu = 1, 2, ..., n \tag{4}$$

where $\hat{x}_\nu \in \mathbb{R}^{n,n}$ and $\lambda_\nu \in \mathbb{R}^{n,n}$ are the ν^{th} eigenvector and eigenvalue of the modified structure, respectively. It is assumed that the modifications do not alter the order of the system. Eq. (4) can be solved either in the physical space or in the modal one. Most finite element analysis software packages solve this problem in the physical space. To have the eigenvalue problem Eq. (4), in modal coordinates, we consider the following coordinates transformation:

$$\hat{x}_\nu = X p_\nu \tag{5}$$

where p_ν is the generalized coordinates vector. Equation (5) implies that the eigenvector $\hat{x}_\nu$ is written as a linear combination of the n eigenvectors of the original structure. Injecting Eq. (5) in Eq. (4) and pre-multiplying by X^T, we obtain the modal representation of the eigenvalue problem of the modified structure:

$$[\Lambda + X^T \Delta K X - \hat{\lambda}_\nu (I + X^T \Delta M X)] p_\nu = 0 \tag{6}$$

where the orthogonality properties of the modes and shapes in Eq. (3) are used to eliminate the mass and the stiffness matrices of the original structure. The solutions of the previous equation are exact if all the modes of the original structure are used. However, this condition is seldom checked, and the solutions can be only approximate. Hence, the first advantage of the modal representation is that the dynamics of the original structure can be adequately represented by relatively few of its fundamental (lowest frequencies) modes. Moreover, it can be applied in a large variety of experimental and analytical cases. Therefore, our goal is to evaluate the eigensolutions of the modified structure without recourse to an exact and, thus, costly reanalysis. In other words, we would like to search for the best approximate solutions of Eq. (6) by using only the identified modal parameters of the original structure. To distinguish between the identified (subscript 1) and the unknown (subscript 2) portions of the original structure eigenbasis, we partition the full matrices X, Λ and p_ν as:

$$X = \begin{bmatrix} X_1 & X_2 \end{bmatrix};\ \Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix};\ p_\nu = \begin{bmatrix} p_{1\nu} \\ p_{2\nu} \end{bmatrix} \tag{7}$$

where $X_1 \in \mathbb{R}^{n,n}$ and $\Lambda_1 = diag\{\lambda_\nu;\ \nu = 1, 2, \ldots\ldots, m\}$ are assumed to be known, $X_2 \in \mathbb{R}^{n,n-m}$ and $\Lambda_2 = diag\{\lambda_\nu; \nu = m+1, \ldots, n\}$ are unknown $p_{1\nu} \in \mathbb{R}^{m,n}$ and $p_{2\nu} \in \mathbb{R}^{n,n-m}$ m is the number of known modes ($m < n$). With this decomposition, Eq. (7) becomes.

$$\hat{x}_\nu = X_1 p_{1\nu} + X_2 p_{2\nu} \tag{8}$$

In what follows, the es will be of a parametric nature (no size change of the original structure). After using decomposition Eq. (7), Eq. (8) is written as the following solving system:

$$\begin{cases} [\Delta_1 + X_1^T \Delta\hat{K} X_1 - \hat{\lambda}_\nu (I_1 + X_1^T \Delta\hat{K} X_1)] p_{1\nu} = 0 \\ X_2 p_{2\nu} = -\Phi R_1 (\Delta K - \hat{\lambda}_\nu \Delta M) X_1 p_{1\nu} \end{cases} \tag{9}$$

$$\Phi = [I + R_1 \Delta K - \hat{\lambda}_\nu R_1 \hat{M}]^{-1} \tag{10}$$

where $\Delta\hat{K} = \Delta K \Phi;\ \Delta\hat{M} = \Delta M \Phi + (\Delta K - \hat{\lambda}_\nu \Delta M) \Phi R_1 M$ *and* $R_1 = X_2 \Lambda_2^{-1} X_2^T$.

The disadvantages of the established equations are that the m^{th} order eigenvalue problem Eq. (9) is non-linear and the matrices $\Delta\hat{K}$ and $\Delta\hat{M}$ may be complicated and costly to evaluate, because they are functions of the inverse matrix Φ.

The latter one contains explicitly the mass matrix M of the original structure and will be calculated for every eigenvalue in the frequency range of interest and for each structural modification. The method

employed for solving the general problem Eq. (9) requires that, in an interest frequency band $[0, f_{max}]$, where f_{max} must be lower or equal to the greatest identified eigenfrequency of the original structure, the matrix Φ is approximated by an $n \times n$ polynomial matrix of n degree:

$$\Phi \cong \Psi[I + \hat{\lambda}_\nu R_1 \hat{M} + \hat{\lambda}_\nu (R_1 \hat{M} \Psi) + \hat{\lambda}_\nu^2 (R_1 \hat{M} \Psi)^2 + ... + \hat{\lambda}_\nu^n (R_1 \hat{M} \Psi)^n] \tag{11}$$

$$\Psi = [I + R_1 \Delta K]^{-1} \tag{12}$$

$$[\Lambda_1 + X_1^T \Delta \breve{K} X_1 - \hat{\lambda}_\nu (I_1 + X_1^T H_1 X_1) - \hat{\lambda}_\nu^2 (X_1^T H_2 X_1) - ... - \hat{\lambda}_\nu^n (X_1^T H_n X_1)] p_{1\nu} = 0 \tag{13}$$

$$\begin{cases} H_k = H_{(k-1)}.D_1 + ... + H_1.D_{(k-1)} - \Delta \breve{K}.D_k \\ H_1 = \Delta \breve{K} - \Delta \breve{M}.D_1 \quad for\ k = 2, ..., n \end{cases} \tag{14}$$

$\Delta \breve{K} = \Delta K \Psi$; $\Delta \breve{M} = \Delta M \Psi$; $D_k = R_k \Delta \breve{M} - R_{(k+1)} \Delta \breve{K}$ and $R_k = X_2 \Lambda_2^{-k} X_2^T$

$$\hat{x}_\nu = \Psi[I + \hat{\lambda}_\nu C_1 + \hat{\lambda}_\nu^2 C_2 + ... + \hat{\lambda}_\nu^n C_n] X_1 p_{1\nu} \tag{15}$$

$$\begin{cases} C_k = D_1.C_{(k-1)} + ... + D_{(k-1)}.C_1 + D_k \\ C_1 = D_1 \quad for\ k = 2, ..., n \end{cases} \tag{16}$$

4. Structural Modification of Dissipative Structures

4.1. *Proportional viscous damping*

In the equation of motion of the modified dissipative structure Eq. (4), we project $\hat{z}_\nu$ the motion vector in the modal base, X, of the conservative structure associated with the original structure:

$$\hat{z}_\nu = Z p_\nu = Z_1 p_{1\nu} + Z_2 p_{2\nu} \tag{17}$$

We replace Eq. (17) in Eq. (2) and multiply by X^T, and it results in the following system:

$$[\Lambda + Z^T \Delta K Z + \hat{s}_\nu (\beta + Z^T \Delta B Z) + \hat{s}_\nu^2 (I + Z^T \Delta M Z)] p_\nu = 0 \tag{18}$$

$\beta = X^T B X$ is the generalized damped matrix. This symmetric matrix is generally full. A necessary and sufficient condition for its diagonalization is $BM^{-1}K = KM^{-1}B$ (this is assuming that Basile is checked only if the eigenvalues are distinct). In practice, β is not diagonal, [17] highlights that the influence of his extra diagonal terms, on mechanical structure behavior, is generally negligible, except in the case of very close eigenfrequencies. Using decomposition of Eq. (7) and substituting Eq. (2) by Eq. (18), which results in the equations:

$$[\Lambda_1 + Z_1^T \Delta K Z_1 + \hat{s}_\nu (\beta_1 + Z_1^T \Delta B Z_1) + \hat{s}_\nu^2 (I_1 + Z_1^T \Delta M Z_1)] p_{1\nu} + Z_2^T [\Delta K + \hat{s}_\nu \Delta B + \hat{s}_\nu^2 \Delta M] Z_2 = 0 \tag{19}$$

$$Z_1^T [\Delta K + \hat{s}_\nu \Delta B + \hat{s}_\nu^2 \Delta M] Z_1 + [\Lambda_2 + Z_2^T \Delta K Z_2 + \hat{s}_\nu (\beta_2 + Z_2^T \Delta B Z_2) + \hat{s}_\nu^2 (I_2 + Z_2^T \Delta M Z_2)] p_{2\nu} = 0 \tag{20}$$

where: $\Lambda_1 = Z_1^T K Z_1$; $\Lambda_2 = Z_2^T K Z_2$; $I_1 = Z_1^T M Z_1$; $I_2 = Z_2^T M Z_2$; $\beta_1 = Z_1^T B Z_1$; $\beta_1 = Z_1^T B Z_1$. Using Eq. (18) and Eq. (19) we can write:

$$[\Lambda_1 + \delta K + \hat{s}_\nu (\beta_1 + \delta B) + \hat{s}_\nu^2 (I_1 + \delta M) Z_1^T \Delta S \Phi R_1 \Delta S Z_1] p_{1\nu} = 0 \tag{21}$$

$$Z_2 p_{2\nu} = -[I + R_1 \Delta K + \hat{s}_\nu R_1 \hat{B} + \hat{s}_\nu^2 \hat{M}]^{-1} R_1 \Delta S Z_1 p_{1\nu} \tag{22}$$

where: $\delta K = Z_1^T \Delta K Z_1; \delta M = Z_2^T \Delta M Z_2; \delta B = Z_1^T \Delta B Z_1; \Delta S = \Delta K + \hat{s}_\nu \Delta B + \hat{s}_\nu^2 \Delta M.$

$$\Phi = [I + R_1 \Delta K + \hat{s}_\nu R_1 \hat{B} + \hat{s}_\nu^2 \hat{M}]^{-1} = \Psi[I - P + P^2 - ...] \tag{23}$$

where: $\Psi[I + R_1 \Delta K]^{-1}; R_1 = Z_2 \Lambda_2^{-1} Z_2^T$ *and* $P = \hat{s}_\nu R_1 (\hat{B} + \hat{s}_\nu \hat{M}) \Psi$

If $\| P \| \ll 1$ (often confirmed thanks to the matrix P form) and if our studies are limited to the developing of $\hat{s}_\nu^2$, Eq. (21) can be reduced to:

$$[A_0 + \hat{s}_\nu A_1 + \hat{s}_\nu^2 A_2] p_{1\nu} = 0 \tag{24}$$

where: $A_0 = [\Lambda_1 + Z_1^T \Delta \hat{K} Z_1]; A_1 = [\beta_1 + Z_1^T (\Delta \hat{K} R_1 \hat{B} \Psi - \Delta \hat{K} R_1 B - \Delta \hat{B}) Z_1]$ *and* $A_2 = [I_1 + Z_1^T \{ \Delta \hat{K} R_1 (\hat{M} \Psi - \hat{B} \Psi R_1 \hat{B} \Psi + \hat{B} \Psi R_1 B - M) + \Delta \hat{B} R_1 (\hat{B} \Psi - B) - \Delta \hat{M} \} Z_1]$ where: $\Delta \hat{M} = \Delta M \Psi; \Delta \hat{K} = \Delta K \Psi$ *and* $\Delta \hat{B} = \Delta B \Psi$

When the proportional damping is a Rayleigh type, matrix B can be written as a weighted sum of the mass and the stiffness matrices (Basile hypothesis), and: $B = \alpha M + \chi K$; α and χ are random coefficients. With this damping, A_1 and A_2 do not explicitly depend on K, M and B.

4.2. *Non-proportional viscous damping*

The modal base of the associated conservative structure does not allow the damping matrix diagonalization B. It is, therefore, preferable to transform the problem with eigenvalues of the second-order of the n dimension into a linear classical form of $2n$ dimension. This modification is achieved by Duncan's transformation, the quantities of generalized movements $M\dot{q}$ of the system, considered as auxiliary variables [18], or by adding a trivial matrix equation, based on generalized velocities [19]. The addition according to the second step of the auxiliary identity is described as follows:

$$M\dot{q}(t) = M\dot{q}(t) \tag{25}$$

$$(A - S_\nu U) y_\nu = 0 \tag{26}$$

$A = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}$ and $U = \begin{bmatrix} B & M \\ M & 0 \end{bmatrix}$ (0 is a n order 0 matrices).

Modal and spectral matrices are grouped in the following forms:

$$\tilde{Y} = \begin{bmatrix} Y & \bar{Y} \\ YS & \bar{Y}\bar{S} \end{bmatrix} \; ; \; \tilde{S} = \begin{bmatrix} S & 0 \\ 0 & \bar{S} \end{bmatrix} \tag{27}$$

where : $\tilde{Y}$, $\tilde{S}$, U and A satisfy the orthogonality constraints $\tilde{Y}^T U \tilde{Y} = \tilde{N}$ and $\tilde{Y}^T A \tilde{Y} = \tilde{N}\tilde{S}$; $\tilde{N} = \begin{bmatrix} N & 0 \\ 0 & \bar{N} \end{bmatrix}$ $N = diag\{ n_\nu \}$; n_ν is a real or a complex scalar, for the following, we will consider $n_\nu = 1$. By decomposing Y and S in a known part Y_1, S_1 an unknown part Y_2, S_2 as:

$$Y = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix} \; ; \; S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \tag{28}$$

where : $Y_1 \in \mathbb{C}^{n,m}$; $S_1 = Diag\{S_\nu = -a_\nu \omega_\nu + j\omega_\nu, \nu = 1, 2, ..., m\}$; $Y_2 \in \mathbb{C}^{n,n-m}$ *and* $S_2 = Diag\{S_\nu = -a_\nu \omega_\nu + j\omega_\nu, \nu = m+1, ..., n\}$

We rewrite $\tilde{Y}$ $\tilde{S}$ as:

$$\tilde{Y} = \begin{bmatrix} \tilde{Y}_1 & \tilde{Y}_2 \end{bmatrix} \; ; \; \tilde{S} = \begin{bmatrix} \tilde{S}_1 & 0 \\ 0 & \tilde{S}_2 \end{bmatrix} \tag{29}$$

where : $\tilde{Y}_1 = \begin{bmatrix} Y & \bar{Y}_1 \\ Y_1 S_1 & \bar{Y}_1 \bar{S}_1 \end{bmatrix}$; $\tilde{Y}_2 = \begin{bmatrix} Y_2 & \bar{Y}_2 \\ Y_2 S_2 & \bar{Y}_2 \bar{S}_2 \end{bmatrix}$;
$\tilde{S}_1 = \begin{bmatrix} S_1 & 0 \\ 0 & \bar{S}_1 \end{bmatrix}$ *and* $\tilde{S}_2 = \begin{bmatrix} S_2 & 0 \\ 0 & \bar{S}_2 \end{bmatrix}$ The changed system has the following equation:

$$[A + \Delta A - \hat{S}\nu(U + \Delta U)]\hat{y}_\nu = 0 \tag{30}$$

If we project the vector $\hat{y}_\nu$ in the modal basis of the initial dissipative structure

$$\hat{y}_\nu = \tilde{Y} c_\nu = \tilde{Y}_1 c_{1\nu} + \tilde{Y}_2 c_{2\nu} \tag{31}$$

And then multiply it by Eq. (30)$\tilde{Y}^T$, we will have:

$$[L + \hat{S}_\nu M + \hat{S}_\nu^2 N]\hat{c}_{1\nu} = 0 \tag{32}$$

where : $L = [\tilde{S}_1 + \tilde{Y}_1^T \Delta A \tilde{Y}_1]$; $M = [I_{2m} - \tilde{Y}_1^T(\Delta U + \Delta A(R_2 \Delta A - R_1 \Delta U))\tilde{Y}_1]$; $N1 = [\tilde{Y}_1^T \Delta A\{R_1(\Delta U R_1 \Delta U - \Delta U R_2 \Delta A) - R_3 \Delta A + R_2(\Delta U - \Delta A R_1 \Delta U + \Delta A R_2 \Delta A)\}\tilde{Y}_1]$; $N2 = [\tilde{Y}_1^T(R_2 \Delta A - R_1 \Delta U)\tilde{Y}_1]$ *and* $N = N1 + N2$; where : $R_1 = \tilde{Y}_2 \tilde{S}_2^{-1} \tilde{Y}_2^T$; $R_2 = \tilde{Y}_2 \tilde{S}_2^{-2} \tilde{Y}_2^T$ *and* $R_3 = \tilde{Y}_2 \tilde{S}_2^{-3} \tilde{Y}_2^T$ We can clearly notice that neither the matrix A nor U appear explicitly.

5. Applications

Two numeric tests are presented to illustrate the proposed method. The test below is used to evaluate the difference between the natural frequencies of the two prototypes of the model: $\delta f_\nu(in\,\%) = \frac{|f_\nu^{ex} - \hat{f}_\nu|}{f_\nu^{ex}} * 100$ $MAC_\nu = [\frac{|Y_\nu^{T^{ex}} - \hat{Y}_\nu|}{||Y_\nu^{ex}||\ ||Y_\nu^{ex}||}]^2$ „ex" is the exact value and "^" represents the approximate value.

5.1. *Example 1: Gantry "with proportional damping"*

In this example, the procedure is applied to gantry Figure 1. This initial model has 74 nodes, each node also has 3 DDL. The modified elements are illustrated in Figure 1 and their modifications are reported in Table 1. The characteristics of the gantry are as follows:

$S_h^i = 0.004m^2\ (section\ horizontal)$
$L_h^i = 15m\ (longueur\ horizontal)$
$S_v^i = 0.006m^2\ (section\ vertical)$
$L_v^i = 3m\ (longueur\ vertical)$
$S_i^i = 0.003m^2\ (section\ incliner)$
$L_i^i = \sqrt{34}m\ (longueur\ incliner)$
$E^i = 0.75e^{11}N/m^2$
$\rho^i = 2800kg/m^3$
$I^i = 0.0756m^4$

Proportional damping of the Rayleigh type on all gantry elements with a damping coefficient a_ν proportional to the mass and the stiffness.

Table 2 presents eigenfrequencies of the initial and the modified structures, calculated directly and with the method [6]. It also compares the $MACs$ of the initial and modified structures.

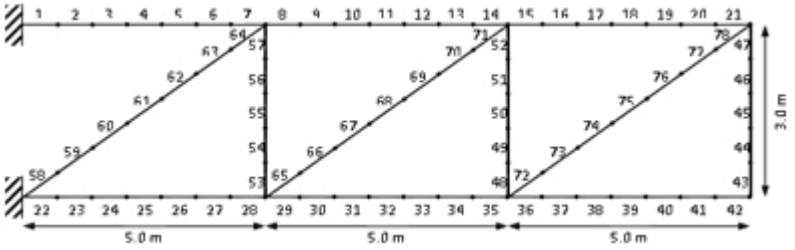

Figure 1. *Gantry portico.*

Table 1. *Parametric changes*

Nb Elements	E^m	I^m	S^m
1	E^i	$10.1 * I^i$	S^i
2	E^i	$10.1 * I^i$	S^i
7	E^i	$1.1 * I^i$	S^i
8	E^i	$1.1 * I^i$	S^i
9	E^i	$1.1 * I^i$	S^i
15	$2.8 * E^i$	$1.1 * I^i$	S^i
17	$2.8 * E^i$	$2. * I^i$	S^i
18	$2.8 * E^i$	$2. * I^i$	S^i
22	E^i	$10.1 * I^i$	$2. * S^i$
23	E^i	$10.1 * I^i$	$2. * S^i$
28	E^i	$10.1 * I^i$	S^i
29	E^i	$10.1 * I^i$	S^i
32	E^i	$1.1 * I^i$	S^i
35	$2.8 * E^i$	$1.1 * I^i$	S^i
40	E^i	I^i	$2.1 * S^i$
41	E^i	I^i	$2.1 * S^i$
43	E^i	$10.1 * I^i$	S^i
46	E^i	$1.1 * I^i$	S^i
51	$2.8 * E^i$	I^i	S^i
52	$2.8 * E^i$	I^i	S^i
55	$1.05 * E^i$	$1.1 * I^i$	S^i
58	E^i	$10.1 * I^i$	S^i
59	E^i	$10.1 * I^i$	S^i
62	E^i	$1.1 * I^i$	S^i
63	E^i	$1.1 * I^i$	S^i
65	E^i	I^i	$2.1 * S^i$
66	E^i	I^i	$2.1 * S^i$
70	E^i	$2 * I^i$	S^i
71	E^i	$2 * I^i$	S^i
72	$1.05 * E^i$	$2.1 * I^i$	S^i
73	$1.05 * E^i$	$2.1 * I^i$	S^i
77	E^i	I^i	$2.1 * S^i$

Figure 2 Shows the results of introducing residual terms R_1. The reference frequencies and the frequencies calculated with and without residues for a structure having proportional damping are plotted. The suggested method has a considerable effect on the minimization of the difference between the exact and calculated frequencies. The same observation can be made on the generalized depreciation differences presented in Figure 5.

Table 2. *The distance between the first six eigenfrequencies of the initial and modified structures calculated exactly and approximatively.*

$MODES$	$f_{\nu_0}^{ex}$	f_{ν}^{ex}	$\hat{f}_{\nu}$	$\delta f_{\nu}(in\ \%)$	MAC_{ν}
1	45.15	52.52	52.52	0.0016	1.0000
2	79.04	84.45	84.45	0.0040	1.0000
3	227.18	236.12	236.22	0.0414	1.0000
4	249.66	289.02	289.11	0.0302	0.9999
5	363.56	377.03	377.22	0.0512	1.0000
6	437.72	452.96	453.14	0.0390	0.9998

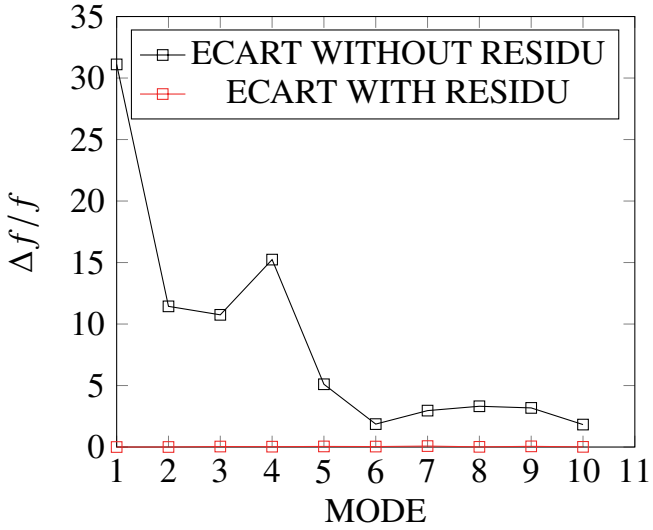

Figure 2. *Frequency with and without residue.*

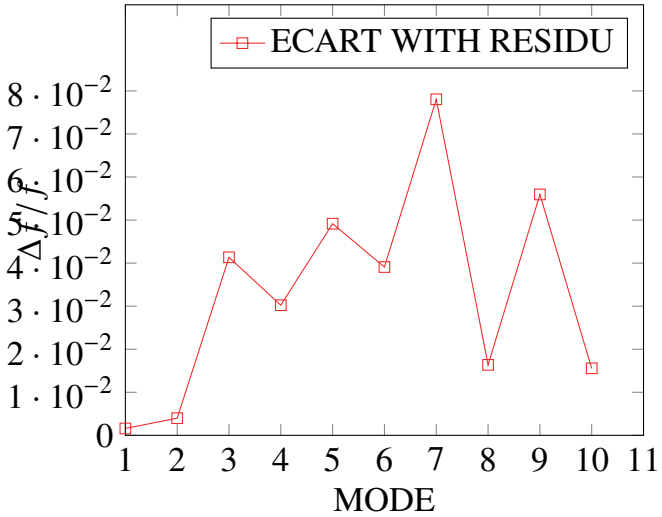

Figure 3. *Frequency with residue.*

5.2. *Example 1: Gantry "with proportional damping"*

The proposed method is applied to a two-dimensional gantry, shown in Figure 6 having 22 finite elements, which have 3 DOFs each. We introduce two damping types: Rayleigh proportional damping, on all frame components with a damping coefficient $a_{\nu} = 2\%$. Nonproportional damping: the initial values

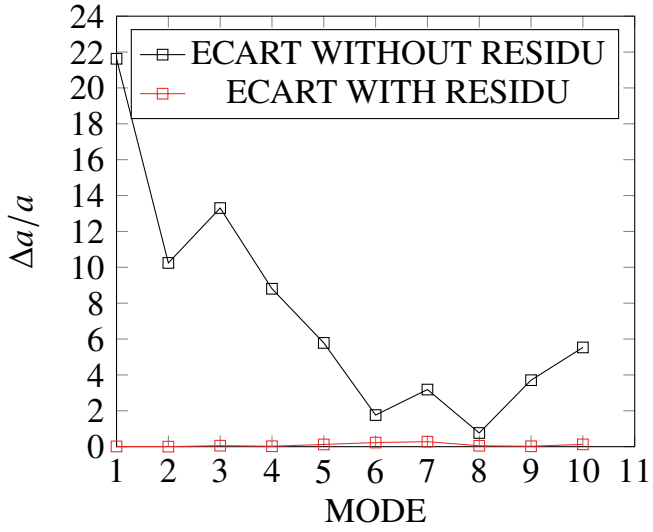

Figure 4. *Frequency with residue.*

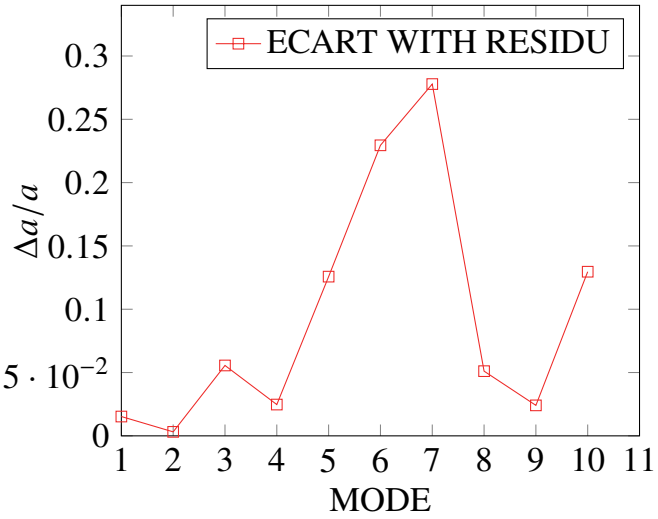

Figure 5. *Frequency with residue.*

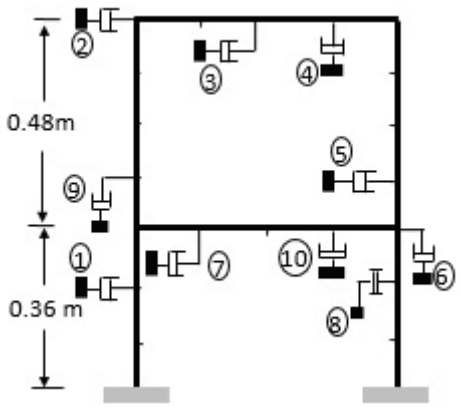

Figure 6. *Initial structure.*

are given in table 3, and both the positions and the investment directions are represented in Figure 6. Tables 4 and 5 plus Figure 7. underline the parametric changes and reports of initial and final disturbed dampers. The initial physical characteristics of the conservative structure are: $Youngmodule: \ E = 2.110^{11} N/m^2$
$Rightsection: \ S = 0.510^{-3} m^2$
$Density: \ \rho = 7800 kg/m^3$
$Inertiamoment: \ I = 0.41710^{-08} m^4$

Starting from the structure in which eigenfrequencies were calculated by [6], and based on the structural modification, we have made a comparison of these calculated eigenfrequencies with our approached method, then we added a nonproportional damping to the structure.

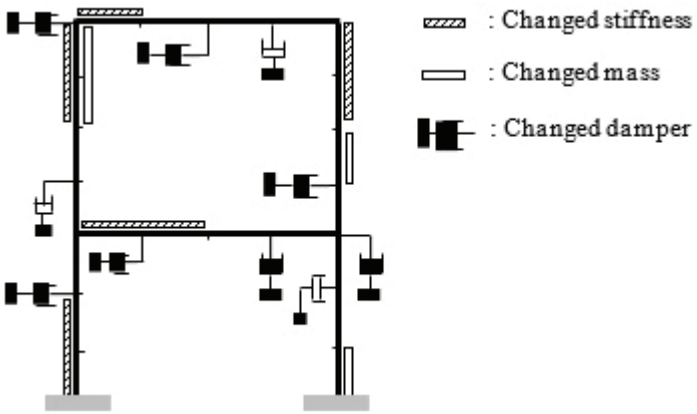

Figure 7. *Changed structure.*

Table 3. *Initial model characteristics*

$Damper\ number$	$Valuekg/s$
1	50
2	70
3	100
4	80
5	150
6	150
7	50
8	100
9	80
10	80

Table 4. *Damper perturbations*

$Damper\ number$	$initial\ value/final\ value$
1	4
3	3
4	2
6	2
7	2
8	2.5
9	2

Table 5. *Density and Young module perturbations*

$Eelement\ Nb$	$\rho\ final/\rho\ initial$	$E\ final/E\ initial$
1	1.	0.5
2	1.	0.5
6	2.	0.5
7	2.	0.5
8	1.	0.5
12	1.	0.5
13	1.	0.5
14	2.	0.5
18	2.	0.5
19	1.	0.5
20	1.	0.5

Table 6. *The distance between the first six eigenfrequencies of the initial and modified structures calculated exactly and approximatively.*

$MODES$	$f_{\nu_0}^{ex}$	f_{ν}^{ex}	$\hat{f}_{\nu}$	$\delta f_{\nu}(in\ \%)$	MAC_{ν}
1	17.96	13.89	13.89	0.000013	0.9999
2	59.71	52.20	52.20	0.000120	0.9999
3	97.56	78.85	78.85	0.000198	1.0000
4	131.15	112.46	112.46	0.000078	0.9999
5	191.53	140.36	140.37	0.000943	0.9999
6	203.62	156.05	156.05	0.002177	1.0000

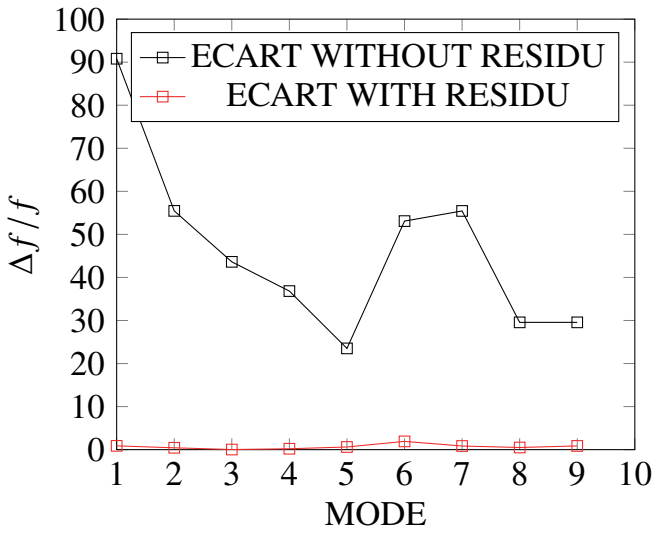

Figure 8. *Frequency with and without residue.*

Figure 8 the relative errors between the approximate and the exact eigenfrequencies of the modified structure are plotted., the difference between the reference frequencies and the frequencies calculated with and without residues for a structure which has nonproportional damping. The suggested method shows that the introduction of the residual terms R_1, R_2 and R_3 has a considerable effect on the minimization of the difference between the exact and calculated frequencies. The same observation can be made on the generalized depreciation differences presented in Figure 10.

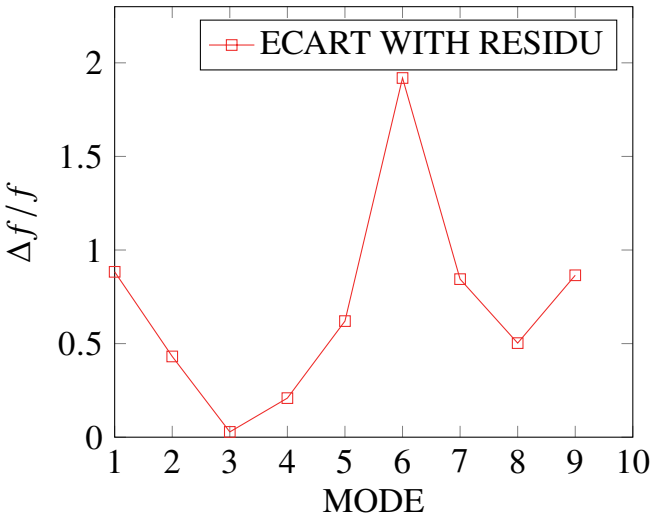

Figure 9. *Frequency with residue.*

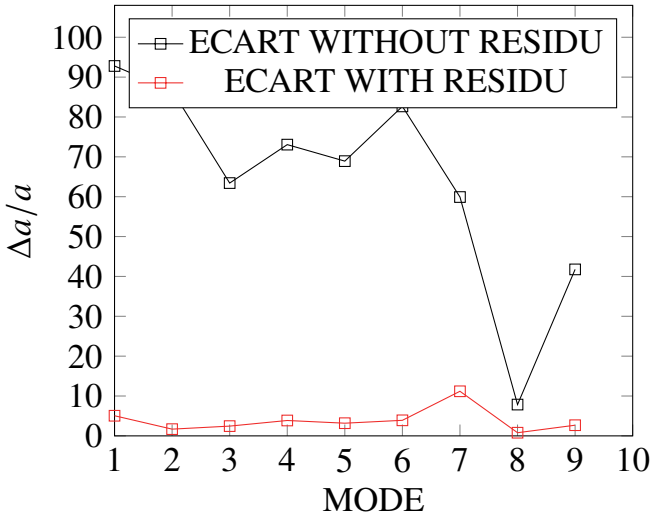

Figure 10. *Damping with and without residuee.*

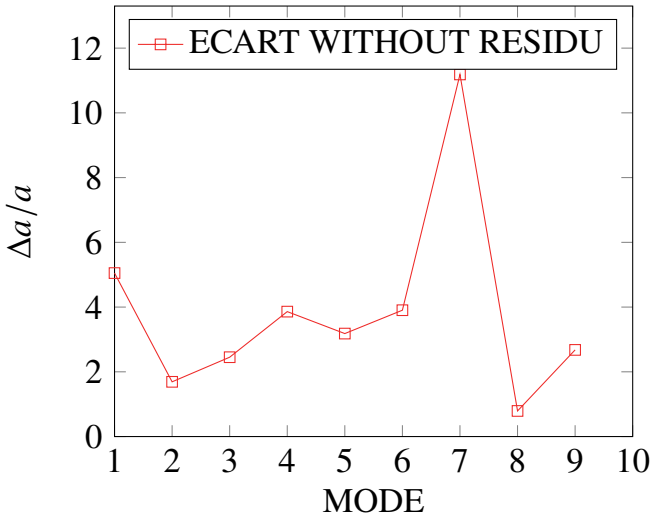

Figure 11. *Damping with and without residuee.*

6. Discussion

This study is based on methods using a projection matrix of the Rayleigh-Ritz type on account of the fact that they are easy to implement and are relatively efficient in terms of precision and operational cost. Adding residues to the truncated modal base helps get an improvement on the calculated solutions as well as the generalized damping. Knowing matrices A and U is not necessary in case we have got data from experimental tests. Regarding the proposed formulations for conservative structures, adding R_1 or (R_1 and R_2) as residues helps to acquire a pleasing result, therefore, using a second order polyeig function does not have any effect on the found precision.

7. Conclusion

This paper focuses on the reanalysis of the structural modification due to the modification of the parameters (mass, stiffness, and damping). A computationally efficient approach for such modifications has been proposed and developed, which involves only the Identification of the spectral sub-basis of the initial structure matrices. In particular, the approach can adaptively control the accuracy of the approximate solutions. Numerical examples have shown that rapid convergence and accurate results can, therefore, be obtained by the proposed approach. This approach can be readily extended to modifications in the boundary conditions, or to substructuring problems.

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