Information-entropy equivalence, Maxwell's demon and the information paradox

L'équivalence information-entropie, le démon de Maxwell et le paradoxe de l'information

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ABSTRACT. Several experiments based on bistable memory have been carried out to confirm Landauer's principle, according to which the erasure of one bit of information requires a minimum energy dissipation of $K_B\ T\ \log 2$. Further experiments on electronic or molecular nanomotors provide a complementary point of view based on a tilt-type memory. By deepening the analysis, we show that their results are sufficiently precise to conclude that, on the contrary, we have to modify the Landauer principle.
Going back to the sources of Landauer's work, we reconsider the machine imagined by Szilard to resolve the paradox of Maxwell's demon, which led von Neumann to propose a form of equivalence between entropy and information. The study of a new variant of Szilard's machine shows that its process does not depend on information held by an observer. In addition, one experiment carrying out the inversion of a bit of information by a quantum process in a tilt memory shows that this inversion can occur without energy consumption, in contradiction with Landauer's principle. The result is a new theoretical approach to solve Maxwell's paradox, according to which a quantum phenomenon also occurs in a bistable memory during the inversion or vanishing of a bit of information, these two transitions showing an average energy dissipation of at least $K_B\ T\ \log 2$ when they are forced by means of a control parameter.
This theory refutes the principle of equivalence between information and entropy which was widely spread under the influence of Shannon and Brillouin, and which is at the origin of the information paradox relating to black holes. The paradox which results from the theories of Bekenstein and Hawking therefore disappears with this principle.
RÉSUMÉ. Plusieurs expériences ont été récemment réalisées pour confirmer le principe de Landauer, selon lequel l'effacement d'un bit d'information requiert une dissipation d'énergie d'au moins $K_B\ T\ \log 2$. Elles reposent sur le principe de la mémoire bistable. D'autres expériences de nano-moteurs électroniques ou moléculaires apportent un point de vue complémentaire basé sur la mémoire à bascule. En approfondissant leur analyse on montre que leurs résultats sont suffisamment précis pour conclure qu'il convient de modifier le principe de Landauer.
En remontant aux sources des travaux de Landauer, nous sommes amenés à reconsidérer la machine imaginée par Szilard pour résoudre le paradoxe du démon de Maxwell, qui a conduit von Neumann à proposer une forme d'équivalence entre l'entropie et l'information. L'étude d'une variante de la machine de Szilard montre qu'en réalité son fonctionnement ne dépend pas de l'information que posséderait un observateur. De plus une expérience réalisant l'inversion d'un bit d'information par un processus quantique en mémoire à bascule montre que cette inversion peut se produire sans dépense d'énergie, en contradiction avec le principe de Landauer. Il en résulte une nouvelle approche théorique pour résoudre le paradoxe de Maxwell, selon laquelle il se produit également un phénomène quantique dans une mémoire bistable lors de l'inversion ou de la disparition d'un bit d'information, ces deux transitions s'accompagnant d'une dissipation d'énergie moyenne d'au moins $K_B\ T\ \log 2$ lorsqu'elles sont forcées au moyen d'un paramètre de contrôle.
Cette théorie réfute le principe d'équivalence entre information et entropie, qui s'est largement propagé sous l'influence des travaux de Shannon et de Brillouin, et qui est notamment à l'origine du paradoxe de l'information relatif aux trous noirs. Ce paradoxe qui résulte de théories de Bekenstein et de Hawking disparait donc avec ce principe.
KEYWORDS. Information, entropy, Szilard machine, Landauer limit, Maxwell's demon, information paradox, bistable memory, tilt memory, tunnelling effect.
MOTS-CLÉS. Information, entropie, machine de Szilard, limite de Landauer, démon de Maxwell, paradoxe de l'information, mémoire bistable, mémoire à bascule, effet tunnel.

1. Introduction

Landauer's principle [Land61] states that the minimum energy to erase a bit of information is $E = K_B\ T\ \log 2$, producing an irreversible increase in entropy $S \geq K_B \log 2$, K_B being the Boltzmann constant (1.380 10 $^{-23}$ J / K) and T the absolute temperature. The erasure of a bit, as stated by Landauer, who defines it as a "*reset to one*", consists in forcing the bit to the value one, whatever its initial value. The principle was confirmed by Bennett [Benn82] and is based on the principle of information-entropy equivalence, proposed by von Neumann [Neum32] to explain the paradox of Maxwell's demon [Knot11]. Over the past ten years or so, several experiments have been carried out in order to confirm Landauer's principle using the concept of bistable memory made of two potential wells. Some of the experiments manipulate a nano-object in Earth's gravity field, moving it sideways from one well to the other by an electric field [DiLu09] [JuGB14] [RMPP14] [GaBe16] or by a viscosity force [BAPC12]. For other experiments, the object is an oscillating cantilever with two zones of stability controlled by an electric field [DPBC21], or a nano-magnet whose stable states are the two preferential orientations of the main magnetic axis, the passage of one to the other being controlled by a transverse magnetic field [HLDB16] [GBMZ17].

An in-depth analysis of these experiences shows that, on the contrary, the Landauer principle should be amended. This prompts us to go back to its foundations, namely Szilard's thought experiment [Szil29] and von Neumann's interpretation of it [Neum32]. A new variant of the Szilard machine proves that its operation is independent of any information or knowledge that the experimenter would have on the state of the system, thus invalidating von Neumann's principle of information-entropy equivalence to solve Maxwell's demon paradox.

These experiments also make it possible to demonstrate that it is not only the erasure of information that causes the consumption of dissipated energy, but also its inversion or its vanishing. Other experiments have been carried out to validate information-entropy equivalence, inspired by the ideas of Maxwell and Szilard. These use a tilt-type memory and show that the inversion of a bit, therefore its erasure in the sense of Landauer, can be done reversibly, without entropy increase. This results in a theorem about the transitions of bistable and tilt memories.

I propose a hypothesis according to which these state transitions cause a quantum phenomenon, identical or analogous to the tunnelling effect, the consequences of which can be measured at the micro or mesoscopic scale. These transitions require an average energy of $K_B\,T\,\log 2$ when they are irreversible, while they can be energetically neutral when reversible.

Shannon [Shan45] [Shan48], inspired by von Neumann, chose to name entropy the probabilistic measure of information that he conceived to solve cryptography problems and to improve the performance of communication systems. He thus indirectly contributed to spreading the principle of information-entropy equivalence, with Brillouin who devoted a book to it [Bril59], having in particular the consequence, for some physicists, of attributing to information a role as fundamental as matter and energy in the physical universe, leading for example to the paradox of information in astrophysics. A brief history of the confusion between information and entropy is given in the Appendix.

2. Maxwell's demon paradox and Szilard's machine

Maxwell had imagined a tiny intelligent demon trying to circumvent the second law of thermodynamics at the molecular level. In an enclosure divided in two by a wall equipped with a tiny diaphragm or valve, the demon filters the passage of molecules by opening the valve to let fast molecules pass in one direction, and slow molecules in the other direction. He would thus transfer heat from a cold zone to a hot zone, without energy consumption, only by the use of his perception and his intelligence.

To resolve this paradox, Szilard [Szil29] designed a system consisting of a single molecule enclosed in a cylindrical box (see state 0 in Figure 1). A mobile valve is suddenly introduced to divide it into two equal volumes (state 1). An observer determines with the help of a measuring device whether the valve has isolated the molecule on the right or the left. They can then use the pressure exerted by the molecule on the mobile valve to recover the energy acquired by the system, namely $K_B T \log 2$, by placing a cable pulled by a weight on the side where the molecule is located (state 2). If the preceding operations could be done without energy consumption, they would contradict the second law of thermodynamics.

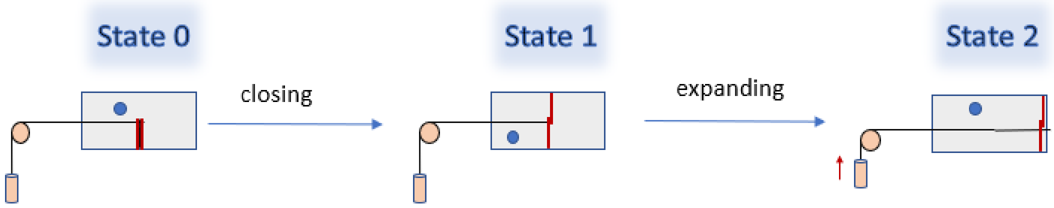

Figure 1. *Szilard's machine*

According to Szilard, it is by measuring the position of the molecule that one expends an energy which then makes it possible to decrease the entropy by $K_B \log 2$. Three years after Szilard's article, von Neumann [Neum32] asserted that it is more precisely the information obtained by the operator by means of the measuring device that allows this entropy reduction, by explicitly establishing (see quotes in the Appendix) an equivalence between the subjective notion of knowledge or information held, in their consciousness, by an observer, and the objective notion of entropy.

We are, however, in the presence of a perfectly defined physical system, regardless of what the operator knows about it. We will study in detail the process by imagining a variant of Szilard's machine, made symmetrical by equipping it with two weights instead of just one (Figure 2).

For a more realistic description, we consider a parallelepiped enclosure, which can be divided into two parts by a transverse valve placed in its middle. The valve consists of two rectangular plates, one of which slides over the other. In the open position they are superimposed, and in the closed position they form an airtight partition in the middle of the enclosure, thus constituting the valve imagined by Maxwell. The valve can slide longitudinally in the enclosure. We fix two cables attached to two weights on either side of the valve, and insert a dynamometer between the end of each cable and the weight to which it is attached. We set up the assembly initially so that the two weights rest on a flat surface, and so that each dynamometer indicates a zero tension force. We note that these dynamometric measurements can be made with an energy approaching zero. To overcome thermodynamic fluctuations, one just has to evaluate the average over a sufficiently long period of time.

The whole system is represented in Figure 2. The numerical values correspond to a nitrogen molecule N_2, whose specific heat ratio is $\gamma = Cp/Cv = 7/5$ at the surrounding temperature T0 = 300 K.

In state 0, the valve is open. Then it is suddenly closed, trapping the molecule in a volume $V1 = V0/2$, which we will assume is on the left (state 1). The molecule undergoes an almost instantaneous adiabatic compression, without heat exchange and therefore without entropy variation, and heats up to a temperature $T1 = T0\ 2^{\gamma-1} = 396\ K$ (123 °C). The internal energy of the molecule that was initially $E0 = \frac{1}{\gamma-1}\ K_B\, T0$ increases by $\Delta E = \frac{2^{\gamma-1}-1}{\gamma-1}\ K_B\, T0 = 0.798\, K_B\, T0$.

Then, in contact with the surrounding medium, the molecule cools down to temperature T0 (state 2), losing an entropy $\Delta S = K_B \log(V1/V0) = -K_B \log 2 = -0.693\, K_B$. There is dissipative heat transfer and this phase of the process is thermodynamically irreversible.

Next, we let the pressure of the molecule push the valve back to one end of the chamber, at very slow speed so that the transformation is quasi-static (state 3). The entropy returns to its initial value S0 and the energy transferred to the weight is equal to $\Delta E = -K_B T0 \log 2$.

We note that the total entropy increase of the system and its environment during the complete cycle is not equal to $K_B \log 2$, but to $\Delta S = \left(\frac{2^{\gamma-1}}{\gamma-1} - \log 2\right) K_B$ or $(0.798 - 0.693) K_B = 0.105\ K_B$.

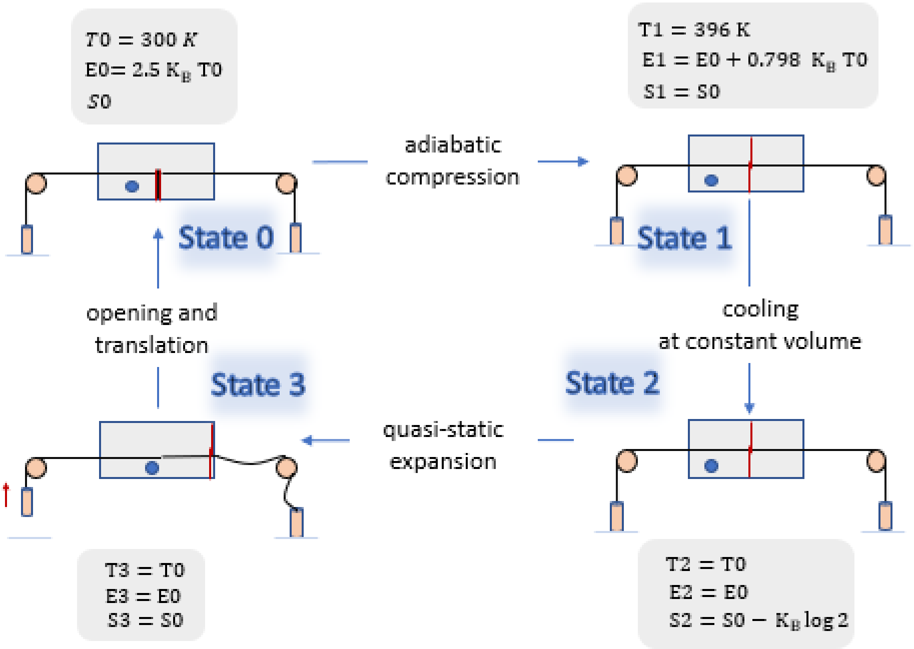

Figure 2. *Symmetric Szilard machine*

Discussion

According to Szilard[1], the measurement of the position of the molecule causes an entropy increase of $K_B \log 2$, balanced by an entropy reduction of the molecule which then allows recovery of an energy of $K_B\ T0 \log 2$. However, our symmetrical variant makes it possible to recover this energy without needing to get the position of the molecule. This invalidates Szilard's hypothesis.

According to von Neumann, it is the fact of knowing on which side the molecule is located that causes an entropy decrease of the molecule (see full citation in the Appendix), which then makes it possible to recover the energy corresponding to this entropy. However, in our symmetrical variant, the operator did not use any information to recover the energy communicated to the system in order to reduce its entropy. No information has changed the entropy of the system. This contradicts von Neumann's hypothesis, and thus refutes his solution to Maxwell's demon paradox.

We must therefore reconsider this paradox. In their arguments, Szilard and von Neumann seem to consider that the closing of the valve is done without providing any energy. However, there is another solution to go from state 0 to state 1. It consists in suddenly moving the valve from one end to the middle of the enclosure (from the right in Figures 1 or 2), against the pressure force of the molecule. We then obtain the same result as we get by closing the valve. This means that by closing the valve we have transferred some energy to the system, so we must have applied a transverse force to the moving part of the valve.

[1] “Taking the measurement is principally connected with a quite certain average entropy production. [...] the entropy produced by this ‘measurement’ must be compensated by the use of the resulting entropy reduction of the system” ([Szil29] p.7).

But where does the force counteracting this transverse force come from? It cannot come from the pressure of the molecule since this is orthogonal to the valve and to its movement. It seems that no theorem of classical mechanics holds the key to this enigma.

3. Landauer's principle and bistable memory

To try and answer this question, we will first deepen the analysis of three experiments which were carried out recently, and which aimed to confirm Landauer's principle.

3.1. *Experiments*

These experiments use the principle of a double potential well which determines two stable states, such that the transition from one state to the other can be controlled by an external action. This physical system makes it possible to represent a bit of information by assigning the symbol 0 to one state and the symbol 1 to the other. It allows us to reverse the bit (NOT operation) by going from one state to the other, or to make it vanish by bringing the potential wells close together until they merge into a single monostable state. The system can therefore be in one of three states: monostable, 0 or 1 (Figure 3).

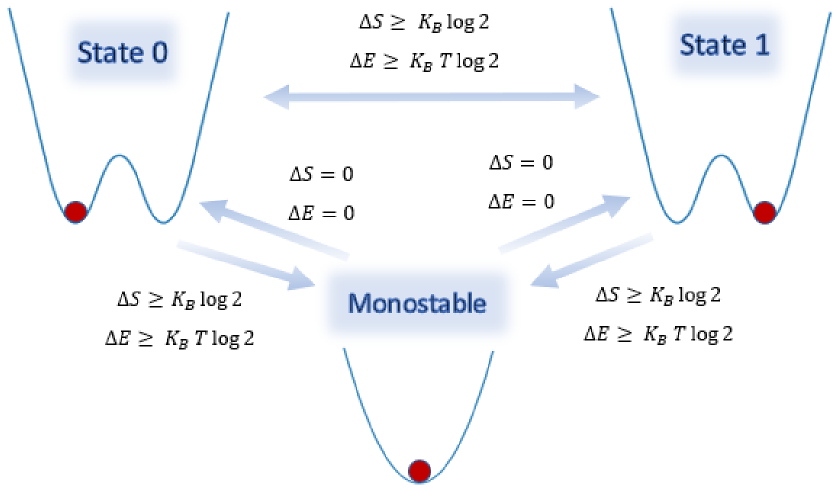

Figure 3. *Transitions of a bistable memory*

<u>Experiment A</u> Gaudenzi et al. [GBMZ17] employ a nanomagnet. The spins of the main magnetic axis can be driven to two opposite directions to make the two potential wells. They apply a cycle with a first phase consisting of canceling the barrier which separates the two potential wells by the action of a transverse magnetic field, which is equivalent to making the information vanish (monostable state). The process results in an average energy dissipation of $K_B T \log 2$ with a good accuracy. But it does not exactly apply the *reset to one* process analyzed by Landauer, which only includes bit inversion and the identity operation, without going through a monostable state. We note that the elementary operation of reversing a spin is a quantum process, which is *a priori* reversible. In the present case, the inversion involves simultaneously a very large number of electron spins (approximately 10^{18} in a 0.4 mg crystal).

<u>Experiment B</u> Dago et al. [DaBe21] carried out an equivalent cycle with a silicon cantilever (mass 2.10^{-13} g) which is made to oscillate by an electric field around two distinct positions, so as to create a double potential well. The initial position is randomly in state 0 or 1. In a first phase, one brings the two potential wells close together until they merge, causing the information to vanish as in the previous experiment. At the end of this phase, an average entropy increase of $K_B \log 2$ is observed in a quasi-static regime with excellent precision. In a second phase, the state 0 is rebuilt, with a

negligible energy consumption compared to $K_B\ T$. So, in this experiment, it is the vanishing of the bit, and only it, which requires energy.

Experiment C   Bérut et al. [BAPC12] were to my knowledge the first to demonstrate experimentally, and with good precision, the energy and entropy involved in the erasure of information. They formed a double potential well using a mechanical-optical system: a tiny glass bead (diameter 2 µm, mass = 10^{-11} g) is brought to an adjustable height by a laser beam which materializes two potential wells separated by a constant distance. A piezoelectric micromotor can apply an increasing viscosity force F to push the bead towards one well or the other in order to change the state. The starting position is distributed randomly between states 0 and 1. During the transition, the force F causes a sudden displacement Δx separating the two potential wells, corresponding to an energy[2] $\Delta E\ =\ F\ \Delta x$ which is irreversibly converted into heat.

Whatever the initial position, one first forces the bead to position 1, then to position 0. It is therefore a *reset to zero*, energetically equivalent to a *reset to one* and matching the process studied by Landauer. The average entropy produced by the erasure is measured over a large number of cycles. The cycle time increases from 5 to 40 seconds to gradually increase the stabilisation time. The mean entropy increase per cycle decreases to a lower limit of about 1.0 K_B for a cycle time of 20 seconds and beyond, which corresponds to a quasi-static regime.

3.2. *Theoretical discussion*

It can be seen that experiment C uses a one-bit erasure method which is different from the other two. One or two bit inversions are carried out, namely two inversions if the bead was initially in position 0 (i.e. 0 => 1 => 0) and only one inversion if the initial position was 1 (i.e. 1 => 1 => 0), in agreement with the cycle studied by Landauer.

In contrast, in the other two experiments, the first phase consists in merging the two potential wells so as to transform the bistable system into a monostable system, before restoring its bistability and re-creating a bit. Experiment B clearly shows, through its excellent precision, that the operation of bit vanishing corresponds to the limit of Landauer from an energy point of view.

In experiment C there is no vanishing of information. The only operations performed are identity (1 => 1) and inversion (NOT: 1 => 0 or 0 => 1). The first is energetically neutral. As the inversion of a bit requires energy and increases the entropy, the experiment makes it possible to evaluate the energy transfer ΔE. For a large number of cycles, since the initial state is evenly distributed between 0 and 1, the mean erasure value is given by ½ (ΔE + 2 ΔE) = 1.5 ΔE.

The experiment gives approximately 1.0 K_B, close to 1.5 log 2 K_B = 1.040 K_B, which contradicts the Landauer principle prediction of log 2 K_B= 0.693 K_B.

In the three experiments the energy consumption is dissipative, leading to an overall entropy increase of the system and its environment. They allow us to conclude that it is the inversion of a bit, as well as the vanishing of a bit, which causes a dissipation of energy of $K_B\ T\log 2$, and not the erasure of a bit in the sense of Landauer.

Where does Landauer's error come from? In his 1961 article, he considers a set of bits to which he applies the *reset to one* operation. He writes: "Consider the extreme case, where the inputs are all ONE, and there is no need to carry out any operation. Clearly then no entropy changes occur and no heat dissipation is involved. Alternatively if the initial states are all ZERO they also carry no information, and no entropy change is involved in resetting them all to ONE."

[2] Typical values for this experiment are F = 3.2 10^{-15} N, Δx = 0.9 µm and ΔE = 2.9 10^{-21} J.

The first statement is correct – the set of bits is not modified, so there is no energy consumption. In contrast, the second is far from obvious. If the transition from 0 to 1 is energetically neutral, then the inverse transition from 1 to 0 must also be, by symmetry.

Moreover, it is wrong to assert that if all the bits are 0 they do not carry any information. Indeed, without compression, a bit carries a quantity of information equal to one. It is the definition of the bit, whether its value is 0 or 1. So *n* bits carry a quantity of information equal to *n*, whatever the value of each of the *n* bits[3], according to Hartley's measure (see Appendix).

Or, if we directly apply the above Landauer reasoning in the case where the set of bits includes only one bit, we then conclude that a bit does not carry any information, whether its value is 0 or 1!

Ultimately, his error stems from the application of the principle of information-entropy equivalence that we refuted above.

4. Tilt memory

The work of Boltzmann and Maxwell led to the creation of statistical thermodynamics, and the thought experiment of Szilard has recently prompted the use of several nanomotors to extract energy from thermal fluctuations. These experiments also use two-state systems, although their goal is not to realise a one bit memory. They also show an entropy variation $\Delta S = K_B \log 2$ during transitions between their two states. We discuss two of these experiments.

Experiment D

Koski et al. [KMPA14] implement a single electron box (SEB) to create an electric current due to quantum tunnelling. The SEB is in state 0 if it contains no electron or in state 1 if it contains one. It is connected upstream to an electron reservoir via a single electron transistor (SET), which allows the passage of electrons one by one via the quantum tunnelling effect. The bistable memories of the previous experiments use two control parameters, the first shaping the two potential wells and the second forcing the transitions from one state to the other. Here a single parameter realises the potential profile which separates the two possible states, and the transitions are spontaneous, due to the tunnelling effect, with a probability depending on the gap ΔE between the energy levels of states 0 and 1.

Figure 4 shows the operation of this tilt memory. It represents the potential profile depending on a voltage applied to the transistor, allowing stabilization of the electron box in one of the two states. When the energy difference approaches zero, the system begins to oscillate between the two states, randomizing the bit of information it contained. If we keep it in this state, we can achieve a new form of bit erasure, different from Landauer's *reset to one* and from the monostable state of experiments A and B.

Experiment D consists of tilting the memory to reverse a bit in quasi-static mode. Spontaneous transitions from one state to another then occur, causing conversions between electrical and thermal energy. Let $\Delta E = E_{post} - E_{ante}$ be the energy difference between the states before and after the transition. If $\Delta E < 0$ some thermal energy is transformed into electrical energy, and vice versa if $\Delta E > 0$. The experiment shows that during the inversion of the bit, the fact of passing from a state 0 or 1 to the state of equilibrium causes the conversion of an average thermal energy $Q = K_B T \log 2$ into electric energy. Then when we go from this state of equilibrium to a state 0 or 1, we get the reverse

[3] A lower value than n can be obtained by measuring the quantity of information, either according to Shannon's formula (section 8), or according to the algorithmic theory of information, but one cannot in any case arrive at a null value.

conversion. This state of equilibrium of the system corresponds to a new form of bit erasure, a randomization, different from *reset to one* or vanishing.

Since erasure, in the Landauer sense, consists of performing one or two bit inversions, this memory can therefore perform it without energy consumption, unlike bistable memory. A logically irreversible operation is therefore carried out physically in a reversible way.

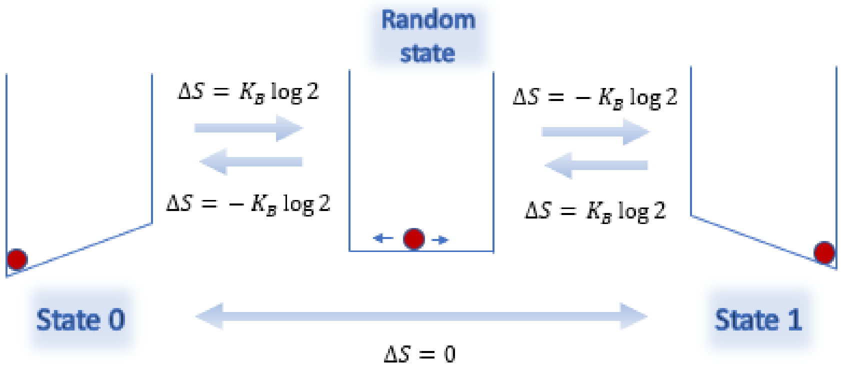

Figure 4. *Tilt memory transitions*

Experiment E

Ribezzi and Ritort [RiRi19] utilize a property of a DNA fragment allowing it to get from a folded state (state 0) to an unfolded state (state 1) depending on the tension force exerted on one end of the molecule. This end is connected through another DNA fragment, functioning as a spring, to a device allowing control of the length of the assembly and to measure the tension force. The operating principle follows the diagram in Figure 4.

This system can produce mechanical energy by thermodynamic fluctuations which cause jumps from one state to the other when their energy levels are close. It also realizes a tilt memory, with a single control parameter, and the measurements have demonstrated the same reversible state transitions as in the previous experiment.

5. New theoretical approach

Since the Szilard and von Neumann theories do not explain the process of Szilard's machine, and since our discussion does not confirm Landauer's principle, we must look for an alternative theory, taking into account the following elements:

- In a Szilard machine, closing the valve causes a sudden energy transfer of $\Delta E = K_B\, T\, \log 2$, which is then partially and irreversibly transformed into heat.
- In a bistable memory, the vanishing or the inversion of a bit causes the irreversible transformation into heat of an energy $\Delta E = K_B\, T\, \log 2$, even during a quasi-static process.
- In a tilt memory, the inversion of a bit is reversible in quasi-static mode, and randomizing one bit reversibly produces an energy of $\Delta E = K_B\, T\, \log 2$.

For these two types of memory it is necessary to use a potential difference between the two states 0 and 1 much higher than $K_B T$, which is the energy level of thermodynamic fluctuations, to ensure the stability of states 0 and 1. The average period of stability of such a system is given by the formula $\tau = \tau_0\, e^{\frac{\Delta E}{K_B T}}$, with τ_0 being a constant [BAPC12].

It should also be noted that in all these experiments the energy measurements show a Gaussian curve with a large standard deviation, which can exceed the mean value by 50%, because of the dispersion of the thermodynamic fluctuations or of the tunnelling effect.

Bistable memory

Let us consider the phase space of a bistable memory, looking only at its two stable states and at the corresponding entropy. In its initial state, whether 0 or 1, the two states are accessible and the phase space contains two configurations. In this case, from the point of view of the Boltzmann equation $S = K_B \log W$, we have W = 2 and the entropy is therefore $S = K_B\, log\, 2$.

If the two states merge into a monostable state (experiments A and B), the phase space is reduced to one configuration, so W = 1 and its entropy becomes $S = 0$. The entropy has therefore varied by $\Delta S = -K_B \log 2$, corresponding to a heat quantity $\Delta Q = -K_B T \log 2$ transmitted to the environment. To do this, it is necessary to provide an equivalent energy $\Delta E = K_B T \log 2$.

If we force the transition from one state to the other (experiment C), the phase space is also reduced to W = 1 at the moment of the transition, since the initial state becomes inaccessible, at least for a certain time. We then find the same entropy variation and the same heat transfer as we did previously.

However, we still have one question to answer. The measurements of experiments A and B show that we can rebuild a bistable state from a monostable state without energy consumption (or more precisely, negligible compared to $\Delta E = K_B T \log 2$). Likewise, once the transition force has been suppressed in experiment C, the bistable state becomes accessible again without energy consumption. However, according to Boltzmann's equation, in these two cases we go from W = 1 to W = 2, corresponding to $\Delta S = K_B \log 2$, and the system should therefore restore the energy $\Delta E = K_B T \log 2$ as happens in a Szilard machine. This is not confirmed by experiments A, B and C. We will come back to this.

Tilt memory

In the tilt memories of experiments D (presence of an electron) and E (molecular bond), the measurements show that the energy exchanges are reversible, without an overall entropy increase. The transitions are spontaneous and are facilitated by bringing the energy levels of states 0 and 1 closer, but they are caused by thermodynamic fluctuations or by the quantum tunnelling effect and not forced by a control parameter as in a bistable memory.

During the bit randomisation in experiment E, we find again the transition from W = 1 to W = 2, and the energy transfer $\Delta E = K_B T \log 2$, corresponding to the application of Boltzmann's theorem. The way back to state 0 or 1 can be done again without energy dissipation.

6. Quantum hypothesis

Let's go back to Szilard's machine. To compress the volume containing the molecule, we can push the valve from the end of the cylinder to its middle, like a piston. The result is the same as if the valve is closed directly in the middle of the cylinder, as in Szilard's thought experiment. But in the latter case classical physics cannot explain how a transverse force can have an effect in an orthogonal direction, that of the axis of the cylinder.

From several previous statements, namely:

– the impossibility to solve the paradox of Maxwell's demon by classical mechanics theorems, in particular in the simplified model of Szilard's machine;

– the quantum phenomena operating directly in experiments A (spin inversion), D (electron tunnelling effect) and E (molecular bond);

– the fact that the entropy in play ($K_B \log 2$) is a constant, independent of the features of the objects used in the experiments (mass of the bead, size of the magnet, mass or frequency of the vibrating plate, energy of the electrons).

I propose the assumption that all these transitions are quantum events, thermodynamically irreversible when they are forced (bistable memories and Szilard machine), and reversible when they are spontaneous (tilt memories).

In the experiments, the transitions of the bistable memories are forced, but it is possible to design a bistable memory whose transitions are spontaneous. We would just have to act on the potential barrier by lowering it to allow a spontaneous transition, and to raise it back as soon as the transition is detected. Indeed, according to modern theory, it is the act of forcing a transition that causes the phenomenon of quantum decoherence, which by nature is irreversible. The action of forcing puts an end to the superposition of two states, as any measurement does in a quantum system.

This may explain why Boltzmann's formula does not apply after the inversion of a bit in a bistable memory (experiment C). At that moment, after the forcing has reduced the phase space from W = 2 to W = 1, the system naturally ends up at W = 2 without further action and without discontinuous modification of the wave function of the system. The same phenomenon occurs when we rebuild a bistable state from the monostable state (experiments A and B), or from a randomized state (experiment E), to re-create a bit of information.

We still have to explain how the equations of quantum physics apply in the mesoscopic mechanical systems of experiments B (glass beads) and C (vibrating plate).

In any case, the entropy and the energy involved in the transitions of these memories are very low[4], much lower than those of current computer memories.

7. Transition theorem and information-entropy relation

A bistable memory is a physical system which has two stable states separated by a potential barrier, such that one can obtain a transition from one state to the other either spontaneously or forced by a control parameter. It is possible to merge the two states into a monostable state, which makes the information vanish.

A tilt memory is a physical system which has two stable states separated by a potential difference and such that, by acting on this difference, the probability of spontaneous transition from one state to the other is modified. In this way, the state of the memory can be reversed.

The barrier or the energy difference which separates the two stable states must be much higher than $K_B T$ to avoid unwanted transitions.

Transition theorem:

1) A bistable memory undergoes a sudden entropy increase of mean value $K_B \, log\, 2$ when it is forced to leave a bistable state, either towards the other bistable state or towards a monostable state. An energy dissipation occurs with an average at least equal to $K_B T \, log\, 2$, K_B being the Boltzmann constant (1.380 10^{-23} J / K) and T the absolute temperature, the equality being achievable in quasi-static mode.

[4] ΔE = 2.9 10^{-21} J at 300 K, and ΔS = 0.96 10^{-23} J / K. The temperature variations are generally also very small. For example, the bead in experiment C (mass 10^{-11} g) heats up by 0.36 10^{-9} K at each transition.

2) The inversion of a tilt or bistable memory by spontaneous transition can be done reversibly, without global entropy increase.

One can imagine an n-stable memory, which has n stable states and such that the transition from one state to another can be controlled at any time. It has a bifurcation point common to the n states, allowing passage from any state to any other state. After experimental verification or demonstration by quantum physics, we should be able to generalize the previous theorem. Going from an n-stable state to another n-stable state through the bifurcation point, the system would undergo a quantum jump of entropy greater than or equal to $K_B \log n$, with an energy transfer of at least $K_B\ T\ \log n$.

Physical reversibility and logical reversibility

In his 1961 article, Landauer [Land61] states that a device is "logically irreversible if its output does not uniquely define its inputs", and he adds that "logical irreversibility implies physical irreversibility, which implies dissipative effects". He further asserts that a reversible computer can only use reversible logical operations which are limited to the identity, the negation, the *exclusive or* and its negation. All this is equivalent to asserting that a device is physically reversible if and only if it is logically reversible, provided that the history of the operations is not memorized, which would obviously allow them to be carried out in reverse.

From the previous results we can conclude that the principle should be amended as follows:

The erasure (in the Landauer sense of *reset to one* or *reset to zero*) or vanishing operations of a bit, which are logically irreversible, are physically irreversible when they are forced, and reversible when they are spontaneous.

The bit inversion, which is logically reversible, is physically irreversible when it is forced, and reversible when it is spontaneous.

Non-equivalence of information and entropy

Single bit operations	**creation**	**inversion**	**erasure (*reset to one*)**	**vanishing randomization**
Bistable memory	$\Delta S = 0$	increase $\Delta S = K_B \log 2$	increase $\Delta S = 1.5\ K_B \log 2$	increase $\Delta S = K_B \log 2$
Tilt memory	$\Delta S = 0$	$\Delta S = 0$	$\Delta S = 0$	transfer $\Delta S = K_B \log 2$

Table 1. *Average entropy increase or transfer ΔS per operation on a physical bit*

The discussion about Maxwell's paradox led us to refute von Neumann's solution which proposed a form of equivalence between information and entropy.

Table 1 summarizes the different types of correspondence between entropy and bit of information which result from the above discussion.

- The inversion or vanishing of a bit in a bistable memory proceeds with an overall entropy increase of $\Delta S = K_B \log 2$, and the *reset to one* operation, equivalent on average to 1.5 inversions, causes 1.5 times this value.

- The creation of a bit can be achieved without entropy increase in a bistable memory. For a tilt memory, the creation, inversion and vanishing of a bit are reversible and can be done with a zero average entropy increase.

- The randomization of a bit in a tilt memory allows transfer from the environment of $\Delta S = K_B \log 2$, in a reversible manner.

The table shows that there is no general relationship between information and entropy, and that we must therefore give up the principle of equivalence between them.

8. Information and entropy – Shannon's formula H

The principle of information-entropy equivalence was proposed for the first time by von Neumann [Neum32]. It is today mostly accepted, although not unanimously. Its history is briefly detailed in the Appendix.

Claude Shannon indirectly contributed to its dissemination through his 1948 article on the theory of communication [Shan48]. This article has led to multiple applications in the development of communications and computers, which does not need to be questioned here. However, we can discuss his use of the term entropy, suggested to him by von Neumann, to define a probabilistic measure of the amount of information, by analogy with the Boltzmann equation. He defines the entropy of information by the formula $H = -K \sum_{i=1}^{n} p_i \log p_i$, which gives the average quantity of information carried by a symbol belonging to a code comprising n symbols, each of which have a probability of occurrence p_i. If the symbols are equiprobable, we find Hartley's formula $Ha = K \log n$ (see Appendix). By taking $K = 1/\log 2$, the unit of measurement is the bit, for a code limited to two symbols.

The value of Shannon's formula is to give the minimum volume of information to which a message can be compressed by using an adapted transcoding. Actually, it is possible to obtain a higher compression rate by taking into account the probabilities of symbol sequences. Shannon himself studied this possibility [Shan51]. An n-gram being a sequence of n symbols, if we calculate by statistics on a given corpus the probability of the presence of the n^{th} symbol as a function of the (n-1) preceding ones, we can greatly reduce the volume of information per symbol. Shannon's results for the English language, based on a given corpus and using the English alphabet of 26 characters, give the following values per symbol: unigram 4.14 bits (corresponding to Shannon's formula above), bigram 3.56 and trigram 3.3, which can be compared to the Hartley formula result $\log_2 26 = 4.70$ bits.

This clearly shows that Shannon’s formula H does not have the character of universality that is often attributed to it. Moreover, the choice of the word entropy to designate this measure has largely contributed to creating confusion between information and entropy due to the suggestion that they are equivalent.

9. The information paradox – the nature of information

Jacob Bekenstein [Beke19] wondered in 1973 about what happens to the information contained in the matter which is absorbed by a black hole. He considers that the entropy of a system or of a certain quantity of matter is equivalent to the information necessary to fully describe the system (by the speed and the angular momentum of each of its particles). According to his theory, if this information is lost, since nothing can come out of a black hole, it contradicts the quantum unitarity principle. It is what he calls the information paradox, which gave rise to debates, in particular with Stephen Hawking, over several decades.

For his part, John Wheeler, who directed Bekenstein's thesis, proposed the slogan "it from bit" to express, according to him, the universal character of information [Zure90] at the same level as mass and energy, if not even more fundamental, to explain the nature of the physical world.

The refutation of the equivalence between information and entropy wipes out these speculations. The information paradox is no more than a question about what happens to all matter that is absorbed by a black hole, and to its entropy. If it is an artefact carrying information such as a book, a lapidary inscription or a computer memory, it is very likely that the enormous pressures inside a black hole destroy their material support and make it impossible to access any stored information. But that doesn't mean that this information is destroyed; any single copy remaining anywhere is enough for it to be preserved.

It is worth recalling that all our previous discussions about the link between information and entropy only concern the materialization of information in the form of physical systems with two states to represent a bit, and not the information itself, which is an intangible entity. Moreover, this only represents a subset, and historically a very recent one, of the information manipulated by animals and humans as coded signals of all forms: the dance of bees, cries of vervet monkeys, human speech, writings drawn on the flat surface of a material or projected onto a screen, etc.

The information came from biological evolution and not from the mineral world [Argo20]. It was created first in the form of RNA and DNA molecules, in the sense that they carry a code and have the property of self-replication, then in the form of animal proto-languages (bees, vervet monkeys, etc.), and finally by the human faculty of language.

10. Conclusion

The solution of Maxwell's demon paradox, proposed by Szilard and von Neumann, was based on an equivalence between the objective concept of entropy and the subjective concept of information or knowledge. The study of a simple variant of Szilard's machine shows that information plays no role, invalidating the solution of Maxwell's paradox conceived by von Neumann, as well as the principle of information-entropy equivalence.

This principle has led Landauer to propose a limit on the energy cost for erasing a bit of information in the case of bistable memory. Recent experiments carried out to validate this limit finally demonstrate that, on the contrary, it is necessary to amend it in the sense that it is the inversion and vanishing of a bit that requires energy and not its erasure in the sense of Landauer, and a reversible logical operation may correspond to a physically irreversible operation. Other experiments undertaken to confirm the principle of information-entropy equivalence use a tilt memory, which circumvents the Landauer limit and can be thermodynamically reversible.

The in-depth study of these two types of experiments led us to propose the hypothesis according to which the transitions of these memories unveil their quantum nature. In three of these experiments, concerning a large number of spins, an electron or a molecule, the quantum character of the transitions is not surprising. For the two other experiments, carried out at a mesoscopic level, it still needs to be made clear how the equations of quantum physics can explain them.

This leads us to looking at Maxwell's demon paradox from a new perspective. The experiments actually reveal two types of Maxwell's demons. A proactive demon, who forces state transitions in a one-bit memory and the closing of the valve of Szilard's machine, and a reactive demon, who detects a spontaneous change of state, due to thermal fluctuations or to quantum tunnelling, and who makes it possible to transform thermal energy in a nanomachine. This reactive demon should allow use of a bistable memory in a quasi-reversible way by controlling the height of the potential barrier to invert the bit of information without energy input.

The principle of information-entropy equivalence led Shannon to define the entropy of information, which is only one of the possible measures of a quantity of information and has the disadvantage of depending on the statistics of its context. The questionable choice of the term entropy to designate this pseudo measure has largely contributed to spreading this principle of equivalence, ultimately to consider information as the third component of the physical world after energy and matter, if not as the very origin of the entire physical world including matter, energy and gravity.

Appendix – A brief history of the information-entropy confusion

The emergence of the concept of information in its modern meaning followed two parallel paths, between physicists and engineers, which von Neumann as a mathematician, physicist and engineer helped to converge.

Harry Nyquist (1889–1976), after completing a PhD in physics, worked at AT&T then at Bell Laboratories to optimize telegraphic transmission. He was the first to quantify the messages transmitted by Morse code. One of the objectives of his work [Nyqu24] consisted in choosing codes allowing transmission of a maximum quantity *of intelligence*[5] depending on the type of signal used. It considers electrical signals coded by several potential levels, up to 16 levels. He calls W the transmission speed of *intelligence* and m the number of possible values of the signal or code used, and proposes the formula $W = K \log m$, with K being a constant. He is thus the first to introduce logarithms to quantify information, which Hartley and Shannon will later adopt and refine.

American engineer Robert Hartley (1888–1970), who worked at Western Electric and then at Bell Laboratories like Nyquist, takes a major step forward in his paper “Transmission of information” [Hart28] where he considers all types of analog signals such as voice, image, television or digital signals such as Morse code or the alphabetical encoding of a text. Without explicitly defining the information, he presents it as a sequence of symbols, communicated from a transmitter to a receiver. If the code is made up of s symbols, he defines the amount of information in a message of n symbols with the formula $H = n \log s$.

Hartley applies his formula to the Baudot code, which its inventor Emile Baudot (1845–1903) developed in 1877 to increase the speed of transmission by teleprinter. Each character is coded on a perforated tape by a transverse line of 5 possible perforations, which gives a code of 5 binary digits, making it possible to represent 2^5 or 32 characters. To increase the number of possibilities for less frequent characters, including digits 0 to 9, special characters and punctuations, we can use two consecutive lines of perforations. The technique has been employed for several decades through several international standards. It allowed for physical storage of the messages transmitted in the form of the perforated tapes. Hartley shows that a row of 5 possible perforations contains a quantity of information of 5 log 2, and extends his analysis to speech, made up of a sequence of words, which he considers to be secondary symbols with the letters as primary symbols.

The following year, in 1929, the physicist Leo Szilard (1898–1964) answered the paradox of Maxwell's demon with his thought experiment, described in section 2. He attributes the energy that allows the decrease of entropy in the system to the measurement of the position of the molecule in the cylinder.

John von Neumann (1903–1957) shared the Hungarian origins of Szilard, who he met again in Berlin during his studies and then in the United States. He published his book, *Mathematical Foundations of Quantum Mechanics* [Neum32], which constitutes an important stage in the history of quantum physics. He devotes a section of his book to Szilard's experience and explains for the first

[5] The word is to be taken here in the sense of *information* (cf. CIA = Central Intelligence Agency).

time the idea of a link between entropy and information (or knowledge, which he uses as a synonym for information in the text).

He writes (p.260): “For a classical observer, who knows all coordinates and momenta, the entropy is therefore constant, and is in fact 0 [...] since these correspond to the highest possible state of knowledge of the observer, relative to the system”, and later “L. Szilard has shown that one cannot get this ‘knowledge’ without a compensating entropy increase K ln 2. In general, K ln 2 is the ‘thermodynamic value’ of knowledge which takes the form of distinguishing between two alternative cases”.

These two sentences contain in essence all the confusion between entropy and information (or knowledge) which will later thrive. To make sure that this is not a transcription error, we can proofread the lines which precede them, concerning Szilard's experiment:

> We have exchanged our knowledge for the entropy decrease K ln 2. Or: the entropy is the same in volume V as in volume V/2, provided we know, in the first-mentioned case, in which half of the enclosure the molecule is to be found. Therefore, if we knew all the properties (position and momentum) of the molecule before the diffusion process was initiated we could calculate at each subsequent moment whether it is on left or right, and entropy would not have changed. If, however, the only information at our disposal were the macroscopic information that the molecule was initially in the right (or left) half of the enclosure, then entropy would increase upon diffusion.

Von Neumann therefore clearly writes that the entropy of a system, which is supposed to be an intrinsic physical property of the system, depends on the knowledge that an observer about it, based on the information at their disposal. Think about the absurdity of the situation in the case where the observer suddenly has a loss of consciousness or perhaps dies: is the entropy suddenly changed at this very moment?

The book by von Neumann has long been a resource used in quantum mechanics. Today however, it is difficult to support the subjective conception of an objective physical quantity, which crept into the paragraph quoted above.

Robert Shannon (1916–2001), after his studies in electrical engineering, which he completed with a highly regarded master's thesis on the application of Boolean algebra to electrical relay circuits, completed a PhD thesis entitled “An algebra for theoretical genetics”. Then, in 1940, he joined the *Institute for Advanced Study of Princeton* where he happened to collaborate with John von Neumann. During the Second World War, we find him at Bell Labs where he devoted himself to cryptography and is where he met Alan Turing. He synthesized his research on cryptography in a classified report entitled “A mathematical theory of cryptography” [Shan45], which was recently declassified. Shannon deals with “the case of discrete information, where the information to be enciphered consists of a sequence of discrete symbols, each chosen from a finite set. These symbols may be letters in a language, words of a language, or amplitude levels of a ‘quantized’ speech or video signal, etc. but the main emphasis and thinking has been concerned with the case of letters”. Nowhere else in his publications does he give a more precise definition of information, which is presented here as a sequence of discrete symbols chosen from a finite set.

He proposes that we measure the information, which he assimilates to a choice, to an uncertainty, by the formula $H = -K \sum_{i=1}^{n} p_i \log p_i$ with the following comment: “quantities of the type $\sum p_i \log p_i$ have appeared previously as measures of randomness, particularly in statistical mechanics. Indeed the H in Boltzmann's H theorem is defined in this way, p_i being the probability of a system being in cell i of its phase space. Most of the entropy[6] formulas contain terms of this type". And later: "The base which is used in taking logarithms in the formula amounts to a choice of the unit of measure. If the base is 10 we will call the resulting units 'digits'; if the base is two the units will be called *alternatives*."

It was Shannon's 1948 publication *A Mathematical Theory of Communication* [Shan48] that gave birth to information theory. It deals with problems of coding messages transmitted by different types of electrical and radio signals, and in the presence of interfering signals, in order to optimise the flow rate of a communication channel. He quotes the works of Nyquist and Hartley from which he borrows the formula $H = n \log s$, and develops the probabilistic approach by $p_i \log p_i$, insisting on the fact that it is necessary to leave aside any semantic aspect of messages: "The semantic aspects of communication are irrelevant to the engineering problem."

He takes up the argument of his classified article, but with no other reference to thermodynamics. It seems that he never asserted that the entropy of information is identical or equivalent to the thermodynamic entropy. We find there the first appearance of the word *bit*: "the choice of a logarithmic base corresponds to the choice of a unit for measuring information. If the base 2 is used the resulting units may be called binary digits, or more briefly *bits*, a word suggested by J.W. Tukey."

He demonstrates in this article that his formula *H* corresponds to the best coding for transmitting information when we know the probabilities p_i . The formula has been the starting point for most works on data compression, but has also been applied out of its domain of validity.

As a first limitation, it relies on the probabilities of using symbols independently of the preceding ones. However, in the same article Shannon shows that the frequency of use of a letter depends on the letters that precede it. He evokes the possibility to use the conditional probabilities of "digrams", of "trigrams" and of "n-grams", which allow better compression of information than formula *H* (see section 8).

The second limitation comes from its statistical basis. The value of *H* depends on the corpus of texts on which the statistics of frequencies are calculated. However, the frequency of words and characters depends on the type of text, place and time taken as reference, since the average vocabulary depends on these parameters. This frequency has no absolute character.

For these two reasons, Shannon's formula *H*, which is almost universally used today to measure the quantity of information, does not have the characteristics of a measure.

In 1948, a few months after Shannon, Norbert Wiener (1894–1964) proposed an equivalent measure of information [Wien48], presented in the form of the integral of a probability function, with the comment: "The quantity we here define as amount of information is the negative of the quantity usually defined as entropy in similar situations." He adds that this idea comes from a personal communication with von Neumann.

Léon Brillouin (1889–1969), a French physicist who emigrated to the United States in 1940 after having been a professor at the College de France, published in 1959 a book entitled *La science et la théorie de l'Information* [Bril59] in which he develops and extends the ideas of Shannon and Wiener. He begins by defining information as a mathematical entity $I = K \log P$ (we find again the Hartley formula), *P* being the number of possibilities of an event. Then he uses Shannon's formula *H* to define the measure of information in the case of *a priori* probabilities p_i of symbols: $I/symbol = -K \sum_{i=1}^{n} p_i \log p_i$.

[6] This is the first and only appearance of the word entropy in this post.

He includes in his book the results of Shannon's 1945 study, stating that the above formula gives "an exact value of the average information per letter", resulting in 4.76 bits / letter for unigrams, 4.03 for digrams, 3.32 for trigrams and 3.1 for 4-grams. Note that these numbers are different from those cited in section 8. This is because the first series took into account the English alphabet of 26 letters, while this one includes the space character in the calculation (both are borrowed from Shannon's article). So he himself illustrates the relative nature of the measure.

To deepen the link with entropy, which Shannon had only mentioned, he analyzed Szilard's experiment and showed the possibility of measuring the speed of a molecule by an optical method which respects Szilard's condition of an increase in entropy of $K_B \log 2$. Then, relying on von Neumann's interpretation, he asserts that negative entropy (which he calls "negentropy") can always be transformed into information, and vice versa. He writes that "Entropy measures the lack of information; it gives us the total amount of missing information on the ultramicroscopic structure of the system." In a later article [Bril64] he adds: "The knowledge of additional information allows us to define more precisely the state of a system, to decrease its number of possible states, and to decrease its entropy. *Any new information increases the negentropy of a system*."[7]

Myron Tribus (1921–2016), an American researcher from MIT, confirms the theories of Shannon and Brillouin [Trib71]: "The conceptual connection between information and the second law of thermodynamics is now firmly established. [...] Information and energy are inextricably interconnected." However, he reports what he calls a joke from von Neumann which he invites us to take seriously:

> In 1961 I asked Shannon what he had thought about when he had finally confirmed his famous measure. Shannon replied: "My greatest concern was what to call it. I thought of calling it 'information', but the word was overly used, so I decided to call it 'uncertainty'. When I discussed it with John von Neumann, he had a better idea. Von Neumann told me 'You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, no one knows what entropy really is, so in a debate you will always have the advantage.'

Rolf Landauer (1927–1999) outlined in 1961 the principle which bears his name, stating that the minimum energy to erase a bit of information is E = $K_B\, T\, \log 2$. He infers it from the Boltzmann equation, and by admitting the entropy-information equivalence of von Neumann and Brillouin. In addition, he shows that it is not the measurement process that requires this energy, but erasure as an irreversible physical process, and that more generally an irreversible logical operation corresponds to an irreversible physical operation.

John Wheeler (1911–2008), one of the great physicists of the 20th century, referred directly to Brillouin in the 1990s when he invented the slogan "*it from bit*". In the introduction to a collective book *Complexity, Entropy and Physics of Information* [Zure90] he explains this slogan: "Otherwise put, every *it* – every particle, every field of force, even the spacetime continuum itself, – derives its function, its meaning, its very existence entirely – even if in some contexts indirectly – from the apparatus-elicited answers to yes-or-no questions, binary choices, *bits*." He writes elsewhere [Whee00]: "Information may not be just what we *learn* about the world. It may be what *makes* the world."

Jacob Bekenstein (1947–2015) devoted his PhD thesis in 1974 to black holes, carried out under the supervision of Wheeler. Among other things, he is interested in what happens to information when it is absorbed by a black hole. He shares the view of von Neumann and Brillouin [Beke19]: "The entropy of

[7] Emphasized by Brillouin.

a system measures one's uncertainty or lack of information about the actual internal configuration of the system”, or “It is possible for an exterior agent to cause a decrease in the entropy of a system by first acquiring information about the internal configuration of the system.” He asks the question:

> We imagine that a particle goes down a Kerr black hole. As it disappears some information is lost with it. [...] We expect the blackhole entropy, as the measure of inaccessible information, to reflect the loss of the information associated with the particle by increasing by an appropriate amount. How much information is lost together with the particles? The amount clearly depends on how much is known about the internal state of the particle, on the precise way in which the particle falls in, etc.

Stephen Hawking (1942–2018), during the debate on the paradox of information, adopts the principle of information-energy equivalence, as shown by the two following quotes:

> A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole [Hawk76].

and

> Entropy can be regarded as a measure of the disorder of a system, or equivalently, as a lack of knowledge of its precise state [Hawk16].

Refutations of the information-entropy equivalence

A number of physicists have disputed this equivalence. A detailed study of their argumentation can be found in [Leff90], who quotes Denbigh (1988), and in [Thim12] who notably reports quotes from Ter Haar (1954), Mandelbrot (1961), Goldsmith (1998), Mirowski (2002) and Pullen (2005). But these authors lacked experimental data to definitively refute the equivalence between information and entropy that goes back to Szilard and von Neumann.

Bibliography

[Argo20] ARGOUARC’H, JEAN: *Archéologie du signe- Théorie unifiée de l’information et de la connaissance*. L’Harmattan., 2020

[BAPC12] BÉRUT, ANTOINE ; ARAKELYAN, ARTAK ; PETROSYAN, ARTYOM ; CILIBERTO, SERGIO ; DILLENSCHNEIDER, RAOUL ; LUTZ, ERIC: Experimental verification of Landauer’s principle linking information and thermodynamics. In: *Nature* Bd. 483 (2012), Nr. 7388, S. 187–189

[Beke19] BEKENSTEIN, JACOB D.: Black Holes and Entropy. In: *Jacob Bekenstein* : World Scientific, 2019 — ISBN 9789811203954, S. 307–320

[Benn82] BENNETT, CHARLES H.: The thermodynamics of computation—a review. In: *International Journal of Theoretical Physics* Bd. 21 (1982), Nr. 12, S. 905–940

[Bril59] BRILLOUIN, LÉON: *La science et la théorie de l’information*. Jacques Gabay., 1959

[Bril64] BRILLOUIN, LÉON: Scientific Uncertainty and Information. In: *Academic Press* (1964)

[DaBe21] DAGO, SALAMBÔ ; BELLON, LUDOVIC: Fast is hot: energetics of information erasure and the overhead to Landauer’s bound (2021). — arXiv: 2105.12023

[DiLu09] DILLENSCHNEIDER, RAOUL ; LUTZ, ERIC: Memory Erasure in Small Systems. In: *Physical Review Letters* Bd. 102, American Physical Society (2009), Nr. 21, S. 210601

[DPBC21] DAGO, SALAMBÔ ; PEREDA, JORGE ; BARROS, NICOLAS ; CILIBERTO, SERGIO ; BELLON, LUDOVIC: Information and Thermodynamics: Fast and Precise Approach to Landauer's Bound in an Underdamped Micromechanical Oscillator. In: *Physical Review Letters* Bd. 126 (2021)

[EaNo99] EARMAN, JOHN ; NORTON, JOHN D.: EXORCIST XIV: The Wrath of Maxwell's Demon. Part II. From Szilard to Landauer and Beyond. In: *Studies in the History and Philosophy of Modern Physics* Bd. 30 (1999), S. 1–40

[GaBe16] GAVRILOV, MOMČILO ; BECHHOEFER, JOHN: Erasure without Work in an Asymmetric Double-Well Potential. In: *Physical Review Letters* Bd. 117, American Physical Society (2016), Nr. 20, S. 200601

[GBMZ17] GAUDENZI, ROCCO ; BURZURÍ, ENRIQUE ; MAEGAWA, SATORU ; VAN DER ZANT, HERRE S. J. ; LUIS, FERNANDO: Quantum-enhanced Landauer erasure and storage (2017). — arXiv: 1703.04607

[Hart28] HARTLEY, RALPH: Transmission of information. In: *Bell System Technical Journal* Bd. 7 (1928), Nr. 3, S. 535–563

[Hawk16] HAWKING, STEPHEN: *Black Holes: The Reith Lectures*: Bantam, 2016

[Hawk76] HAWKING, STEPHEN: The Quantum Mechanics of Black Holes. In: *Scientific American* (1976), S. 34–40

[HLDB16] HONG, JEONGMIN ; LAMBSON, BRIAN ; DHUEY, SCOTT ; BOKOR, JEFFREY: Experimental test of Landauer's principle in single-bit operations on nanomagnetic memory bits. In: *Science Advances*, American Association for the Advancement of Science (2016)

[JuGB14] JUN, YONGGUN ; GAVRILOV, MOMČILO ; BECHHOEFER, JOHN: High-Precision Test of Landauer's Principle in a Feedback Trap. In: *Physical Review Letters* Bd. 113, American Physical Society (2014), Nr. 19, S. 190601

[KMPA14] KOSKI, JONNE V. ; MAISI, VILLE F. ; PEKOLA, JUKKA P. ; AVERIN, DMITRI V.: Experimental realization of a Szilard engine with a single electron. In: *Proceedings of the National Academy of Sciences* Bd. 111, National Academy of Sciences (2014), Nr. 38, S. 13786–13789

[Knot11] KNOTT, CARGILL GILSTON: „Quote from undated letter from Maxwell to Tait". Life and Scientific Work of Peter Guthrie Tait. In: *Cambridge University Press* (1911), S. 213–215

[Land61] LANDAUER, RALPH: Irreversibility and Heat Generation in the Computing Process. In: *IBM Journal of Research and Development* Bd. 5 (1961), Nr. 3, S. 183–191

[Leff90] LEFF, HARVEY: *Maxwell's Demon : Entropy, Information, Computing*. Princeton University Press., 1990

[Neum32] VON NEUMANN, JOHN: *Mathematical Foundations of Quantum Mechanics*. Princeton University Press., 1932

[Nyqu24] NYQUIST, HARRY: Certain Factors Affecting Telegraph Speed. In: *Bell System Technical Journal* (1924)

[RiRi19] RIBEZZI-CRIVELLARI, M. ; RITORT, F.: Large work extraction and the Landauer limit in a continuous Maxwell demon. In: *Nature Physics* Bd. 15 (2019), Nr. 7, S. 660–664

[RMPP14] ROLDÁN, É ; MARTÍNEZ, I. A. ; PARRONDO, J. M. R. ; PETROV, D.: Universal features in the energetics of symmetry breaking. In: *Nature Physics* Bd. 10 (2014), Nr. 6, S. 457–461

[Shan45] SHANNON, CLAUDE: A Mathematical Theory of Cryptography (1945)

[Shan48] SHANNON, CLAUDE: A mathematical theory of communication. In: *Bell System Technical Journal* Bd. 27 (1948), Nr. 3, S. 379–423

[Shan51] SHANNON, CLAUDE: Prediction and entropy of printed English. In: *Bell System Technical Journal* (1951)

[Szil29] SZILARD, LEO: On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings. In: *Zeitschrift fur Physik* Bd. 9 (1929), Nr. 4, S. 301–310

[Thim12] THIMS, LIBB: Thermodynamics ≠ Information Theory: Science's Greatest Sokal Affair. In: *Journal of Human Thermodynamics* Bd. 8 (2012), S. 120

[Trib71] TRIBUS, MYRON: Energy and Information. In: *Scientific American* (1971)

[Whee00] WHEELER, JOHN: Geons, Black Holes and Quantum Foam: A Life in Physics. In: *American Journal of Physics* Bd. 68 (2000)

[Wien48] WIENER, NORBERT: *Cybernetics or Control and Communication in the Animal and the Machine*. Reissue Of The 1961 Second Edition. Cambridge, MA, USA : MIT Press, 1948 — ISBN 978-0-262-53784-1

[Zure90] (1990) ZUREK, WOJCIECH: Complexity, Entropy And The Physics of Information. In: *Routledge & CRC Press*