Simulation numérique d'un problème vibroacoustique par synthèse modale

Numerical simulation of vibroacoustic problems by the modal synthesis

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ABSTRACT. In the vibroacoustic studies of coupled fluid-structure systems, reducing the size of the problem is important because we must add all the degrees of freedom of the fluid domain to those of the structure. We propose a modal synthesis method for solving this type of problem, coupling dynamic substructure of Craig and Bampton type and acoustic subdomain based on a pressure formulation. The application of the proposed method is performed on a boat propeller with four blades, in air and water. Our numerical results are confronted to some experimental study which allows validate jointly the calculation process and the proposed method.
KEYWORDS. Fluid-structure interaction; reduction of model; vibroacoustic; modal synthesis.

1. Introduction

Currently multiple problems of coupling fluid-structure arise for example in environmental engineering, in aeronautics, in the transportation industry, and naval industry where the study of the response of coupled systems remains a delicate subject [4,7]. The study of these phenomena requires a vibroacoustic analysis, which resulted the specific scientific research on behavior at frequencies in structures [1,6]. So we understand the importance of establishing in advance of any realization of reliable numerical models to predict such behavior. However, the complexity of the phenomena studied is reflected in the costs of prohibitive computation, so it is possible to build reduced order models for these problems based on reduction techniques developed in the both areas. These techniques will bring writing the systems of small dimensions obtained from the analysis of a classical numerical formulation.

We propose a modal synthesis method for solving this type of problem, coupling dynamic substructure of type Craig and Bampton and subdomain based on an acoustic pressure formulation [3]. Using these methods is justified by both the numerical advantages and the treatment of problems of large dimensions [5].

2. Problem statement

We consider an acoustic fluid in a rigid cavity, in contact with an elastic structure. The variables used to describe the structure Ω and the acoustic cavity F are respectively the displacement u and the pressure p. We note ρf the fluid density, C the speed of sound in the fluid, and ρs, E and ν are respectively the density, Young's modulus and Poisson's ratio of the structure. In the sequel, the indexes s and f shall designate respectively the numbers of substructures and subdomains. Each substructure occupies a volume noted Ωs. Each subdomain fluid occupies a volume Ωf. The structure Ω is composed of Ns substructures Ωs (s=1,...,Ns) and the acoustic fluid F is constituted by Nf acoustic subdomains Ωf (f=1,...,Nf). Into we distinguish three types of interface, defined as follows: $L^{ss'} = \Omega^s \cap \Omega^{s'}$, $L^{ff'} = \Omega^f \cap \Omega^{f'}$ and $L^{sf} = \Omega^s \cap \Omega^f$.

$L^{ss'}$ denotes the interface (or junction) between the substructure Ω^s and the substructure $\Omega^{s'}$. $L^{ff'}$ represents the interface between the fluids subdomains Ω^f and $\Omega^{f'}$ ($L^{ff'} = \emptyset$ in the absence of contact between the two subdomains fluid). L^{sf} is the fluid-structure interface between the substructure Ω^s and the fluid subdomain Ω^f.

We assume that each substructure is modeled by finite elements. In a calculation by dynamic substructuring, the vibration behavior of substructure result of external forces applied to them, and binding forces exerted over them other substructures. In a calculation by dynamic modal synthesis, the vibratory behavior subdomains result of external forces applied to them, and binding forces exerted over them other subdomains. Given the acoustic character of considered problem, we do not take into account here the effects related to gravity.

For the substructure

$$\sigma^s_{ij,j}(u) + \omega^2 \rho_s u^s = F^s_e + F^{s'}_L \text{ in } \Omega^s \quad [1]$$

$$u^s = 0 \text{ on } L^{ss} \quad [2]$$

$$\sigma^{ss}_{ij}(u) n^{ss}_j = F^{ss}_e n^{ss}_i \text{ on } L^{ss} \backslash L^{sf} \quad [3]$$

For acoustic subdomain

$$\Delta p^f + \frac{\omega^2}{c^2} p^f = a^f_e + a^{f'}_L \text{ in } \Omega^f \quad [4]$$

$$p^f = 0 \text{ on } L^{ff} \quad [5]$$

The coupling conditions

$$\sigma^s_{ij}(u) n^s_j = p^f n^f_i \text{ on } L^{sf} \quad [6]$$

$$\frac{\partial p^f}{\partial n^f} = \omega^2 \rho_f u^s n^f \text{ on } L^{sf} \quad [7]$$

n^s is the outward normal of the Ω_s, n^f is the outward normal of the Ω_f, L^{ss} is the border of the substructure, f^s_e is the vector of external forces exerted on the substructure and $f^{s'}_L$ is the vector of the bonding forces exerted by the substructures s' adjacent to s.

Taking into account the boundary conditions [6], the variational formulation of the substructure is:

$$\int_{\Omega^s} \sigma^s{}_{ij}(u^s) \varepsilon^s{}_{ij}(v) d\Omega - \omega^2 \int_{\Omega^s} \rho_s u^s v d\Omega = \int_{L^{ss} \backslash L^{sf}} F^s_e v \, dL + \int_{L^{ss'}} F^{s'}_L v \, dL + \int_{L^{sf}} p^f n^f \quad [8]$$

The variational formulation of acoustic subdomain is:

$$\int_{\Omega^f} \frac{\partial p^f}{\partial x_i} \frac{\partial q}{\partial x_i} d\Omega - \omega^2 \int_{\Omega^f} \frac{1}{c^2} p^f q d\Omega = \omega^2 \int_{L^{sf}} u^s n^f q dL; \quad [9]$$

2.1. *Finite element approximation*

After discretization by finite element method of the variational formulation [8] and in the case of the modal calculation we can write:

$$M^s \ddot{u}^s + K^s u^s = F^s_e + \sum_{\substack{s'=1 \\ s' \neq s}}^{N^s} \{F^{s'}_L\} \quad [10]$$

where: $\{u^s\} = \begin{Bmatrix} u^s_i \\ u^s_j \end{Bmatrix}$ $[M^s] = \begin{bmatrix} M^s_{ii} & M^s_{ij} \\ M^s_{ji} & M^s_{jj} \end{bmatrix}$ $[K^s] = \begin{bmatrix} K^s_{ii} & K^s_{ij} \\ K^s_{ji} & K^s_{jj} \end{bmatrix}$

$\{u^s\}$ the displacement vector of each substructure, $\{u_i^s\}$ the vector of internal degrees of freedom and $\{u_j^s\}$ the vector of degrees of freedom of junction. In complex notation the substructure Ω^s satisfies the following equation:

$$(K^s - \omega^2 M^s)\{u^s\} = \{F_e^s\} + \sum_{\substack{s'=1 \\ s'\neq s}}^{N^s}\{F_L^{s'}\} \qquad [11]$$

After discretization by finite element method of the variational formulation [9] and in the case of the modal calculation we can write:

$$E^f \ddot{P}^f + H^f P^f = a_e^f + \sum_{\substack{f'=1 \\ f'\neq f}}^{N^f}\{a_L^{f'}\} \qquad [12]$$

In complex notation we found:

$$(H^f - \omega^2 E^f)\{P^f\} = \{a_e^f\} + \sum_{\substack{f'=1 \\ f'\neq f}}^{N^f}\{a_L^{f'}\} \qquad [13]$$

The vector $\{P^f\}$ contains all the unknown degrees of freedom associated to the pressure (the degrees of freedom on the boundaries Γ_p^f of imposed pressure known are not contained in these vectors). $[H^f]$ is the mass matrix of fluid subdomain Ω^f, $[E^f]$ the stiffness matrix of the fluid subdomain Ω^f and $\{a^f\}$ the vector of external equivalent pressures. $\{a_L^{f'}\}$ represents the accelerations at the interface between the acoustic subdomain Ω^f and the acoustic subdomain $\Omega^{f'}$. Assembling the equations [11] and [13] gives the algebraic system describing the vibroacoustic problem substructure / subdomain following:

$$-\omega^2\begin{bmatrix} M^s & 0 \\ \rho_f L^{sf^t} & E^f \end{bmatrix} + \begin{bmatrix} K^s & -L^{sf} \\ 0 & H^f \end{bmatrix}\begin{Bmatrix} u^s \\ p^f \end{Bmatrix} = \begin{Bmatrix} F_e^s \\ a_e^f \end{Bmatrix} + \begin{Bmatrix} \sum_{\substack{s'=1 \\ s'\neq s}}^{N^s}\{F_L^{s'}\} \\ \sum_{\substack{f'=1 \\ f'\neq f}}^{N^f}\{a_L^{f'}\} \end{Bmatrix} \qquad [14]$$

$[L^{sf}]$ is the matrix of coupling between the sub-structure Ω^s and the subdomain fluid Ω^f.

2.2. *Modal synthesis*

For the structure

The modal synthesis method is to find the unknown displacement field on an appropriate space of reduced dimension (Ritz transformation). For each sub-structure, this space is composed of dynamic modes and static deformed:

$$u^s = \begin{bmatrix} \varphi_i^s & \psi_j^s \end{bmatrix}\begin{Bmatrix} v_i^s \\ v_j^s \end{Bmatrix} = \phi^s v^s \qquad [15]$$

φ_i^s are the modal vectors associated to dynamic natural modes of sub-structure, ψ_j^s are the modal vectors associated to the static deformed of substructure, v_i^s is the vector of the generalized coordinates associated to natural modes of the substructure, v_j^s is the vector of the generalized coordinates associated to the static deformed of substructure, v^s is the vector of the generalized coordinates of the substructure. The projection of the equation [11] on the basis of the substructure Ω^s taking into account [15] we can write:

$$(\bar{K}^s - \omega^2 \bar{M}^s)\{v^s\} = \{\bar{F}_e^s\} + \sum_{\substack{s'=1 \\ s'\neq s}}^{N^s}\{\bar{F}_L^{s'}\} \qquad [16]$$

Assuming that the dynamic natural modes and static deformed are organized as shown in formula [15] and considering that the natural vectors associated at the dynamic modes are normalized relatively to the modal mass unit. The generalized matrices mass and stiffness take the following form:

$$\bar{M}^s = \begin{bmatrix} I & \varphi^{s^t} M^s \psi^s \\ \psi^{s^t} M^s \varphi^s & \psi^{s^t} M^s \psi^s \end{bmatrix}; \quad \bar{K}^s = \begin{bmatrix} \mu^s & \varphi^{s^t} K^s \psi^s \\ \psi^{s^t} K^s \varphi^s & \psi^{s^t} K^s \psi^s \end{bmatrix}$$

I is the identity matrix and μ^s is the diagonal matrix of the squares of the natural angular frequencies of the base.

On prove in the case of the method of Craig-Bampton, that the normal modes and the constrained modes are orthogonal relative to the stiffness matrix whose extra-diagonal terms are therefore zero. The equation of movement of the entire structure is obtained by combining the N^S substructures in a global vector containing all the degrees of freedom of the structure: $\langle u \rangle = \langle u^1 \; u^2 \; \dots u^{N^S} \rangle$. By projecting on the following equation:

$$\{u\} = [\varphi]\{v\} \qquad [17]$$

where $[\varphi] = \begin{bmatrix} \varphi^1 & 0 \\ 0 & \varphi^{N^S} \end{bmatrix}$ $\{v\} = \begin{Bmatrix} v^1 \\ \vdots \\ v^{N^S} \end{Bmatrix}$.

The assembly of the equation [16] leads to the following algebraic system:

$$(\bar{K} - \omega^2 \bar{M})\{v\} = \{F\} + \{F_L\} \qquad [18]$$

where : $[\bar{M}] = \begin{bmatrix} \bar{M}^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \bar{M}^{N^S} \end{bmatrix}$, $[\bar{K}] = \begin{bmatrix} \bar{K}^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \bar{K}^{N^S} \end{bmatrix}$ $\{\bar{F}\} = \begin{Bmatrix} \bar{F}^1 \\ \vdots \\ \bar{F}^{N^S} \end{Bmatrix}$, $\{\bar{F}_L\} = \begin{Bmatrix} \sum_{s' \neq 1} \bar{F}_L^{1s'} \\ \vdots \\ \sum_{s' \neq N^S} \bar{F}_L^{N^S s'} \end{Bmatrix}$

$[\bar{M}]$ and $[\bar{K}]$ are symmetric. And taking into account the conditions of compatibility at the interface, the continuity of displacements between substructure, the equation [19], and the equilibrium of the interface, equation [20], the last term of equation [17] disappear.

$$\{v_j\}^1 = \{v_j\}^2 = \dots = \{v_j\}^{N^S} = \{v_j\} \qquad [19]$$

$$\sum_{s=1}^{N^S} \{F_j\}^{N^S} = 0 \qquad [20]$$

In fact the degrees of freedom {v} are not linearly independent. The linear relations between these degrees of freedom come from the equality of displacement at interface substructure / substructure. They can be expressed by a global matrix of connectivity [S] ; {w} contains only the degrees of freedom linearly independent. For Craig and Bampton method the matrix [S] is boolean. From equation (19) it is easy to show that this equation implies:

$$[S]^t[\varphi]^t\{F_L\} = \{0\} \qquad [21]$$

Thus, the following system is obtained after the projections of equation [21] on the equation [18]:

$$(\bar{\bar{K}} - \omega^2 \bar{\bar{M}})\{w\} = \{\bar{\bar{F}}\} \qquad [22]$$

where: $[\bar{\bar{K}}] = [S]^t[\bar{K}][S]$, $[\bar{\bar{M}}] = [S]^t[\bar{M}][S]$ and $\{\bar{\bar{F}}\} = [S]^t\{\bar{F}\}$

For the fluid

The physical degrees of freedom for each the fluid subdomain can now be composed on their local modal basis as follows:

$$\{P^f\} = [\psi^f]\{q^f\}, \qquad [23]$$

$[\psi^f]$ is the matrix of the selected modes to complaisant interfaces after truncation. $\{q^f\}$ is the generalized coordinates vector associated to subdomain Ω^f, containing the coefficients associated at modes of complaisant interfaces well as the physical degrees of freedom of fluid junction.

The projection of the equation (13) on the basis of subdomain Ω^f can be writing:

$$(\bar{H}^f - \omega^2 \bar{E}^f)\{q^f\} = \{\bar{a}_e^f\} + [\psi^f]^t \sum_{\substack{f'=1 \\ f'\neq f}}^{N^s} \{a_L^{f'}\} \qquad [24]$$

where : $[\bar{E}^f] = [\psi^f]^t[E^f][\psi^f]$; $[\bar{H}^f] = [\psi^f]^t[H^f][\psi^f]$; $\{\bar{a}_e^f\} = [\psi^f]^t\{a_e^f\}$

Assembling the N^s fluid subdomains in a global vector containing all the fluid degrees of freedom, organized as : $\langle P\rangle = \langle P^1 \; P^2 \ldots P^{N^f}\rangle$. We replace it in equation (25) we find the following algebraic system:

$$(H - \omega^2 E)\{P\} = \{a_e\} + \{a_L\} \qquad [25]$$

where:

$$[E] = \begin{bmatrix} E^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & E^{N^f} \end{bmatrix}, \; [H] = \begin{bmatrix} H^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H^{N^f} \end{bmatrix} \; \{a\} = \begin{Bmatrix} a^1 \\ \vdots \\ a^{N^f} \end{Bmatrix}, \quad \{a_L\} = \begin{Bmatrix} \sum_{f\neq 1} a_L^{1f} \\ \vdots \\ \sum_{f\neq N^f} F_L^{ff'} \end{Bmatrix}$$

The matrices [E] and [H] are symmetric. The local decomposition of equation [23] can be assembled as follows:

$$\{P\} = [\psi]\{q\} \qquad [26]$$

where: $[\psi] = \begin{bmatrix} \psi^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \psi^{N^f} \end{bmatrix}, \; \{q\} = \begin{Bmatrix} q^1 \\ \vdots \\ q^{N^f} \end{Bmatrix}$

$[\psi]$ is the matrix collecting the reduced modal bases of all fluid subdomains. The projection of the equation [26] on the equation [25] leads to the following algebraic system:

$$(\bar{H} - \omega^2 \bar{E})\{q\} = \{\bar{a}_e\} + \{a_L\} \qquad [27]$$

with: $[\bar{E}] = [\psi]^t[E][\psi]$ $\quad [\bar{H}] = [\psi]^t[H][\psi]$ $\{\bar{a}_e\} = [\psi]^t\{a_e\}$

Taking into account the continuity conditions at the interface fluid/fluid (equal acceleration on these interfaces) will be assured by the matrix [T] such that:

$$\{q\} = [T]\{r\} \qquad [28]$$

where {r} contains only linearly independent degrees of freedom. The matrix [T] characterizes the connectivity of fluid subdomains between them, and the more it is Boolean. Taking into account the compatibility conditions at the interface, equation [22], the last term in equation [27] disappear.

$$\{a_L^{f}\} + \{a_L^{f'}\} = \{0\} \qquad [29]$$

We can prove that this equation implies: $[T]^t[\psi]^t\{a_L\} = \{0\}$. Thus, the final system to solve for the fluid domain can be written:

$$\left(\bar{\bar{H}}^f - \omega^2 \bar{\bar{E}}^f\right)\{r^f\} = \{\bar{\bar{a}}_r^f\} \qquad [30]$$

where: $\left[\bar{\bar{F}}^f\right] = [T^f]^t[\bar{F}^f][T^f]$ $\quad \left[\bar{\bar{H}}^f\right] = [T^f]^t[\bar{H}^f][T^f]$ $\quad \{\bar{\bar{a}}_r^f\} = [T^f]^t\{\bar{a}_r^f\}$

Compared to the system [27], this model in practice is greatly reduced because its size is the number of orthogonal local modes retained after truncation, which must be added the total number of junction degrees of freedom.

For the vibroacoustic problem

Structural and fluid degrees of freedom are grouped in a global vector:

$$\langle u \;\; p\rangle = \langle u^1 \; u^2 \; \ldots u^{N^S} \;\; p^1 \; p^2 \ldots p^{N^f}\rangle$$

Taking into account the interaction between the substructure Ω^s and subdomain fluid Ω^f. The global matrix can be written:

$$[L] = \begin{bmatrix} L^{11} & \cdots & L^{1N^f} \\ \vdots & \ddots & \vdots \\ L^{N^S1} & \cdots & L^{N^SN^f} \end{bmatrix}$$

$[L^{sf}]$ are implicitly zero when there is no interface between the substructure Ω^s and the fluid subdomain Ω^f.

The equations (32) below can be assembled into a single equation:

$$\{u\} = [S][\varphi]\{w\}\,,\ \{p\} = [T][\psi]\{r\} \qquad \begin{Bmatrix} u \\ p \end{Bmatrix} = [C][R]\begin{Bmatrix} w \\ r \end{Bmatrix} \qquad [31]$$

where: $[C] = \begin{bmatrix} S & 0 \\ 0 & T \end{bmatrix}$ and $[R] = \begin{bmatrix} \varphi & 0 \\ 0 & \psi \end{bmatrix}$

Finally the vibroacoustic problem reduced by modal synthesis to solve is:

$$[C]^t[R]^t\left(-\omega^2\begin{bmatrix} M & 0 \\ \rho_f L^t & E \end{bmatrix} + \begin{bmatrix} K & -L \\ 0 & H \end{bmatrix}\right)[C][R]\begin{Bmatrix} w \\ r \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad [32]$$

3. Numerical results

3.1. *The propeller*

The propeller is the most important technical element on a boat. Its design and characteristics have a direct influence on performance energy. The propeller transforms the power supplied by the motor for the thrust propelling the boat on water. Looking at the profile and cross section of the four blades, we see a powerful form that will push the boat very well (see Figure 1).

Figure 1. *Propeller with four blades*

3.2. *Numerical results*

Figure 2 shows the finite element model of the whole propeller and Figure 3 shows the finite element model of blade. The fluid and the structure are defined by their properties illustrated in Tables 1 and 2.

E(Pa)	ν	ρ_s (Kg.m^{-3})
9.6×10^{10}	0.3	9200

Table 1. *Material properties of the structure*

Pf (Kg.m^{-3})	c (m.s^{-1})
1000	1500

Table 2. *Material properties of the fluid*

The numerical developments were done in Ansys. The proposed method of the reduction of the model is applied to a simplified model of the propeller made up of four substructures, and the acoustic cavity is subdivided into four subdomains acoustic.

Figure 2. *Finite element model*

The numerical calculations are performed on the whole structure and then on a single blade in air and water, the results are shown in Tables 3, 4, 5 and 6 are compared to experimental results [2].

Modes	ANSYS	EXPERIMENTAL
R1	75.534	73
R2	121.46	117
R3	209.17	201

Table 3. *Natural frequency of the whole propeller in air*

Modes	ANSYS	EXPERIMENTAL
R1	37 .581	36
R2	68.629	65
R3	127.46	123

Table 4. *Natural frequency of the whole propeller in water*

Figures (4) shows the first three modes of the whole propeller in air and water.

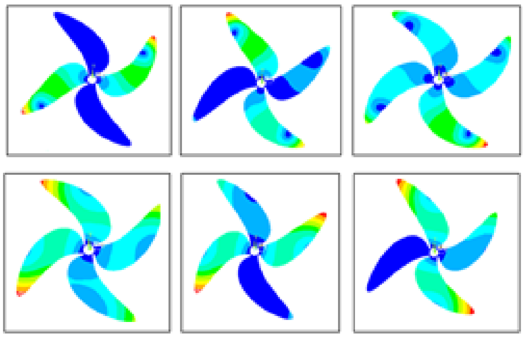

Figure 5. *The first three mode of propeller in air and in water*

The observation of a decrease in frequencies of the propeller in water shows that the effects of acoustic waves on the structure are correctly described by the numerical calculation. Tables 3 and 4 give the results of calculation and measurement in air and water carried on the whole propeller. Tables 5 and 6 show the calculation results in air and water for a single blade of propeller. The numerical calculation gives the comparable results to those obtained by experimentation with an uncertainty compared to that given by other authors [2,8].

4. Conclusion

This work proposes a numerical method for solving large vibroacoustic problem for the coupled fluid-structure systems, modeled by the finite element method. As a first step we established the basic equations of vibroacoustic problem using a displacement/pressure formulation for a substructure and a subdomain. After a finite element discretization, we lead to an algebraic system describing the coupled problem substructure /subdomains. The application of the proposed method is performed on a boat propeller with four blades in air and in water.

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